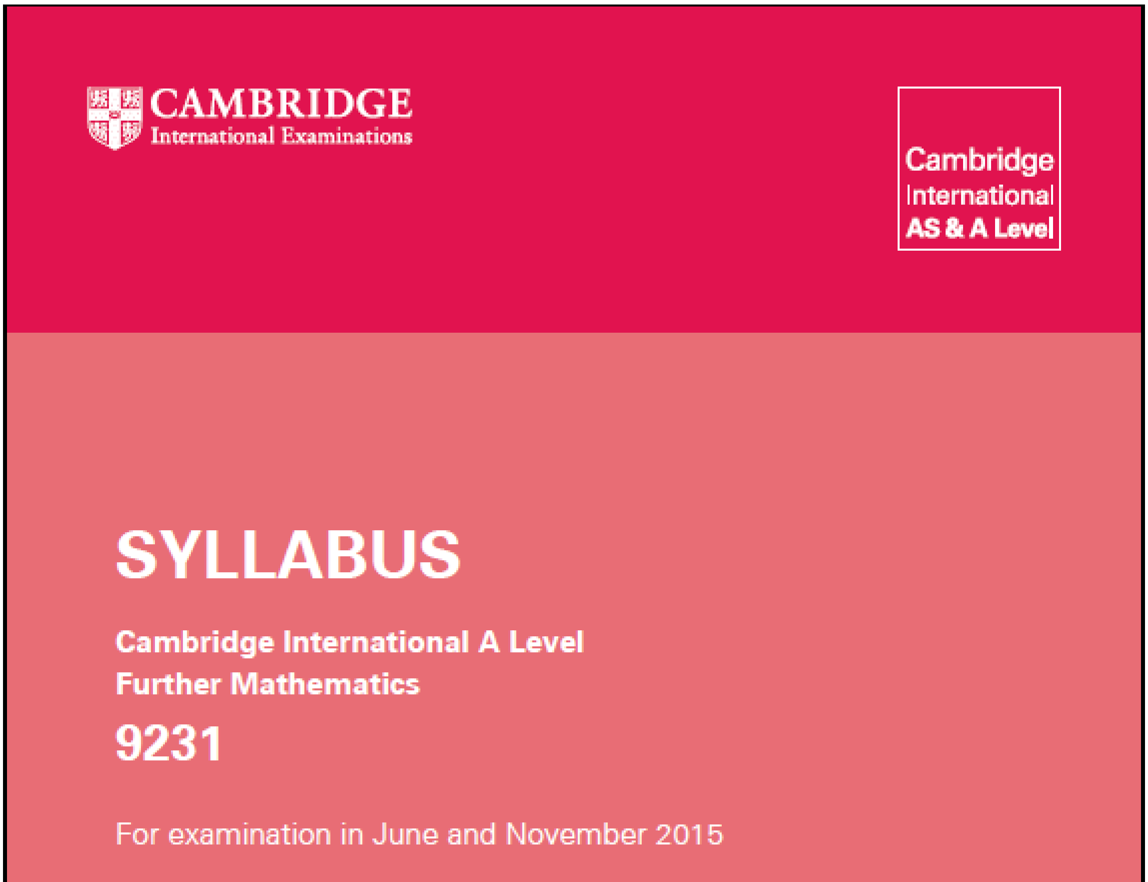
CAMBRIDGE
International Examinations

Cambridge International AS & A Level

SYLLABUS

Cambridge International A Level Further Mathematics

9231

For examination in June and November 2015

Outline proposal for syllabus changes

At Cambridge we have been reviewing a number of our International AS and A Level syllabuses, to ensure that they are up to date with current thinking and continue to give learners world class preparation for higher education or employment.

For our International AS and A Level Further Mathematics we have been consulting teachers, examiners and colleagues in higher education. We are now presenting our proposals in outline in this document and we would like to know what you think of them.

We will be bringing out the revised version of the syllabus for first examination in 2017.

Key

Description of proposed changes to the syllabus

Information on proposed changes

Highlights proposed new content

Learners studying Cambridge International AS and A Levels have the opportunities to:

- acquire an in-depth subject knowledge
- develop independent thinking skills
- apply knowledge and understanding to new as well as familiar situations
- handle and evaluate different types of information sources
- think logically and present ordered and coherent arguments
- make judgements, recommendations and decisions
- present reasoned explanations, understand implications and communicate them clearly and logically
- work and communicate in English.

Guided learning hours

Cambridge International A Level syllabuses are designed on the assumption that candidates have about 360 guided learning hours per subject over the duration of the course. Cambridge International AS Level syllabuses are design ... about 180 guided learning hours per subject over t ... number of hours required to gain the qu ... prior experience of the sy

For this qualification, learners are currently expected to have previous knowledge of M1, M2, S1 and S2. This is more than required applied mathematics units for A level Mathematics. Does this cause Further Mathematics students any problem?

er

C ... employers as proof of math ... skills, including:

- a deeper u
- the further developmen ... ing the use of applications of mathematics in the context of everyday situa ... r subjects that they may be studying
- the ability to analyse probl ... y, recognising when and how a situation may be represented mathematically
- the use of mathematics as ... ns of communication
- a solid foundation for furthe ... udy.

Prior learning

Knowledge of the syllabus for Pure Mathematics (units P1 and P3) in Mathematics 9709 is assumed for Paper 1, and candidates may need to apply such knowledge in answering questions.

Knowledge of the syllabus for Mechanics units (M1 an ... in Mathematics 9709 is assumed for Paper 2. C ... questions; harder questions on those units

Progression

Cambridge International A Level Furth ... Mathematics or related courses in highe

In this section we will introduce the subject and set out key concepts which, if well understood, will contribute to the mastery of mathematics by learners. These may include concepts such as problem solving, mathematical modelling and mathematical reasoning and proof. These key concepts will be shared with A Level Mathematics.

Key consultation questions for A Level Further Mathematics

There are a number of questions to consider for A level Further Mathematics.

We would like to consult which would be the best structure for Further Mathematics, in order to offer centres more flexibility, the best course structure and a positive assessment experience.

There are three options for the structure of A Level Further Mathematics, and for each there are a number of issues to consider. We would like your view on which of these options you prefer, bearing all the issues in mind.

Should there be an AS qualification available in Further Mathematics? If so, which components should be included?

Should the structure be split into four shorter papers?

Is the gap between applied content in 9709 and the applied content in 9231 a barrier to learners?

Currently the expected knowledge for 9231 is the content of P1, P3, S1, S2, M1 and M2. This content is more than what is required for any A level Mathematics qualification, for which only two applied mathematics components are needed.

Should the assumed applied mathematics content for Further Mathematics be M1 and S1 only, which would lead to some content in Further Mathematics overlapping with M2 and S2?
Should the assumed applied mathematics content for Further Mathematics be M1, M2, S1 and S2, as now? This is the “gap” situation described in the three different structure options.

What do these structures look like?

Structure 1 – current 9321 structure

Paper 1 – Pure Mathematics 50% of the A Level	Paper 2 – Applied Mathematics 50% of the A Level

Previous knowledge required: 9709 (P1, P3, M1, M2, S1 and S2)

Structure 2 – splitting the current content into four papers

Paper FP1 – Further Pure Mathematics 1 25% of the A Level	Paper FP2 – Further Pure Mathematics 2 25% of the A Level
Paper FM – Further Mechanics 25% of the A Level	Paper FS – Further Statistics 25% of the A Level

Previous knowledge required: 9709 (P1, P3, M1, M2, S1 and S2)

Structure 3 – splitting the current content into four papers

Paper FP1 – Further Pure Mathematics 1 25% of the A Level	Paper FP2 – Further Pure Mathematics 2 25% of the A Level
Paper FM – Further Mechanics 25% of the A Level (Includes content from M2)	Paper FS – Further Statistics 25% of the A Level (Includes content from S2)

Previous knowledge required: 9709 (P1, P3, M1 and S1)

In structure 3, a student who had studied only mechanics or only statistics in 9709 (either M1 and M2 or S1 and S2) would have some content in 9231 they had met before, but would be required to catch up with the other applied mathematics content.

Structure Option 1

3. Assessment at a glance

All candidates take two papers.

Paper 1	3 hours
There are about 11 questions of different marks and lengths on Pure Mathematics. Candidates should answer **all** questions except for the final question (worth 12–14 marks) which will offer two alternatives, only one of which must be answered. 100 marks weighted at 50% of total	

Paper 2	3 hours
There are 4 or 5 questions of different marks and lengths on Mechanics (worth a total of 43 or 44 marks) followed by 4 or 5 questions of different marks and lengths on Statistics (worth a total of 43 or 44 marks) and one final question worth 12 or 14 marks. The final question consists of two alternatives, one on Mechanics and one on Statistics. Candidates should answer **all** questions except for the last question where only one of the alternatives must be answered. 100 marks weighted at 50% of total	

Structure of 9231 Further Mathematics

The first option is to retain the structure as it is: two papers, equally weighted, one covering Pure Mathematics and one covering Applied Mathematics.
This would allow one AS Level route, taking the Pure Paper only.
There would be a gap in applied mathematics content between 9709 and 9231.
There would be no optional questions on any of the papers.
There would be some changes to the content to make it more coherent.

Removing the optional questions from the papers will ensure everyone has the same assessment experience.

Structure Option 2

Structure of 9231 Further Mathematics

The second option is to split the current content into four equally weighted units. There would be two units of Further Pure Mathematics and two units of applied mathematics: Further Mechanics and Further Statistics.
This would allow three AS level routes.
There would be a gap in applied mathematics content between 9709 and 9231.
There would be no optional questions on any of the papers.
There would be some changes to the content to make it more coherent.

Structure Option 3

Structure of 9231 Further Mathematics

The third option is to split the current structure into four equally weighted units. There would be two units of Further Pure Mathematics and two units of applied mathematics: Further Mechanics and Further Statistics.
This would allow there to be three AS level routes available.
The content for Further Mechanics and Further Statistics would extend the content of Papers 5 and 7 of A Level Mathematics 9709. For this structure, the assumed prior learning of applied mathematics content would be M1 and S1.
There would **no gap** in applied mathematics content between 9709 and 9231.
There would need to be further consultation to decide what happens with learners who have studied only mechanics or statistics in 9709.
There would be no optional questions on any of the papers.
There would be some changes to the content to make it more coherent.

Assessment at a Glance for these structures is on the next page.

3. Assessment at a glance (Option 2 and Option 3)

AS Level Further Mathematics
All candidates take two papers.

Paper FP1 **1 hour 30 minutes**	**Paper FP2** **1 hour 30 minutes**
There are about 6 questions of different marks and lengths on Further Pure Mathematics 1. Candidates should answer **all** questions 50 marks weighted at 50% of total	There are about 6 questions of different marks and lengths on Further Pure Mathematics 2. Candidates should answer **all** questions 50 marks weighted at 50% of total

OR

Paper FP1 **1 hour 30 minutes**	**Paper FM** **1 hour 30 minutes**
There are about 6 questions of different marks and lengths on Further Pure Mathematics 1. Candidates should answer **all** questions 50 marks weighted at 50% of total	There are about 6 questions of different marks and lengths on Further Mechanics. Candidates should answer **all** questions 50 marks weighted at 50% of total

OR

Paper FP1 1 hour 30 minutes	Paper FS 1 hour 30 minutes
There are about 6 questions of different marks and lengths on Further Pure Mathematics 1. Candidates should answer **all** questions 50 marks weighted at 50% of total	There are about 6 questions of different marks and lengths on Further Statistics. Candidates should answer **all** questions 50 marks weighted at 50% of total

A Level Further Mathematics
All candidates take four papers.

Paper FP1 1 hour 30 minutes	Paper FP2 1 hour 30 minutes
There are about 6 questions of different marks and lengths on Further Pure Mathematics 1. Candidates should answer **all** questions 50 marks weighted at 50% of total	There are about 6 questions of different marks and lengths on Further Pure Mathematics 2. Candidates should answer **all** questions 50 marks weighted at 50% of total
Paper FS **1 hour 30 minutes**	**Paper FM** **1 hour 30 minutes**
There are about 6 questions of different marks and lengths on Further Statistics. Candidates should answer **all** questions 50 marks weighted at 50% of total	There are about 6 questions of different marks and lengths on Further Mechanics. Candidates should answer **all** questions 50 marks weighted at 50% of total

Electronic Calculators

Candidates should have a calculator with standard 'scientific' functions for use in the examination. Graphic calculators will be permitted but candidates obtaining results solely from graphic calculators without supporting working or reasoning will not receive credit. Computers, and calculators capable of algebraic manipulation, are not permitted. All the regulations in the *Cambridge Handbook* apply with the exception that, for examinations on this syllabus only, graphic calculators are permitted.

Mathematical Instruments

Apart from the usual mathematical instruments, candidates may use flexicurves in this examination.

Removing graphical calculators

The proposal is to allow scientific calculators only in the examinations. This is to ensure all candidates have the same sort of assessment experience. Graphical calculators could still be used for teaching and learning activities.

Mathematical Notation

Attention is drawn to the list of mathematical notation at the end of this booklet.

Mathematical notation

The proposal is to consider removing any mathematical notation that isn't used in either 9709 or 9231.

Formula books

The proposal is combine the formula books for 9709 and 9231. This will encourage progression for learners who study both syllabuses.
Some formulae may be added to reflect any changes to the content.

4. Syllabus aims and assessment objectives

4.1 Syllabus aims

There are no plans to change the aims of the syllabus.

The aims for Advanced Level Mathematics 9709 apply, with appro

The aims are to enable candidates to:

- develop their mathematical knowledge and skills in a way which encourages confidence and provides satisfaction and enjoyment
- develop an understanding of mathematical principles and an appreciation of mathematics as a logical and coherent subject
- acquire a range of mathematical skills, particularly those which will enable them to use applications of mathematics in the context of everyday situations and of other subjects they may be studying
- develop the ability to analyse problems logically, recognise when and how a situation may be represented mathematically, identify and interpret relevant factors and, where necessary, select an appropriate mathematical method to solve the problem
- use mathematics as a means of communication with emphasis on the use of clear expression
- acquire the mathematical background necessary for further study in this or related subjects.

4.2 Assessment objectives

The assessment objectives for Advanced Level Mathematics 9709 apply, with appropri

The abilities assessed in the examinations cover a single area: **technique with a** examination will test the ability of candidates to:

- understand relevant mathematical concepts, terminology and notation
- recall accurately and use successfully appropriate manipulative technique
- recognise the appropriate mathematical procedure for a given situation
- apply combinations of mathematical skills and techniques in solving problems
- present mathematical work, and communicate conclusions, in a clear and logica

Splitting the current text will help clarify the Assessment objectives. There is no fundamental change in meaning.

Assessment objectives

The sub bullets in the assessment objective will be split into two separate assessment objectives.

AO1 Knowledge and techniques

- Understand relevant mathematical concepts, terminology and notation
- Recall accurately and use successfully appropriate manipulative techniques

AO2 Mathematical problem –solving

- Recognise the appropriate mathematical procedure for a given situation
- Apply combinations of mathematical skills and techniques in solving problems
- Present mathematical work, and communicate conclusions, in a clear and logical way.

Syllabus content changes

There will be some changes to the current Further Pure content. How this content is arranged and ordered will depend on which structure option is used.
The content of these two new units would be based on the existing 9231 content.
The two proposed units, FP1 and FP2, relate to structure options 2 and 3.
If structure option 1 is used, the same content changes are proposed but this would be presented and assessed as a single paper.

The previous knowledge required will be clarified.

The mathematical content for each unit in the scheme is detailed below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

As well as demonstrating skill in the appropriate techniques, candidates will be expected to apply their knowledge in the solution of problems. Individual questions set may involve ideas and methods from more than one section of the relevant content list.

Proposed Further Pure Mathematics 1 content (FP1)

Further Pure Mathematics 1 content

About half of the existing Pure Mathematics content from A Level Further Mathematics 9231 would be used to form the first pure unit, FP1. This unit would rely upon the knowledge and understanding from the A Level Mathematics. This unit would be compulsory for any AS Level course and the A Level course. There would be no optional questions on this paper.

Theme or topic	Curriculum objectives
1. Polynomials and rational functions	*Candidates should be able to:* • recall and use the relations between the roots and coefficients of polynomial equations, for equations of degree 2, 3, 4 only; • use a given simple substitution to obtain an equation whose roots are related in a simple way to those of the original equation; • sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes).

This section will be renamed to reflect the content better, and more clarity regarding graph sketching has been included

1. Algebra and graphs	• recall and use the relations between the roots and coefficients of polynomial equations, for equations of degree 2, 3, 4 only; • use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation; • use the standard results for Σr, Σr^2, Σr^3 to find related sums; • use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions; • recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases; • sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with the axes; **determination of the set of values taken by the function, e.g. by the use of a discriminant, is included).**

2. Polar coordinates	• understand the relations between cartesian and polar coordinates (using the convention $r \geqslant 0$), and convert equations of curves from cartesian to polar form and *vice versa*; • sketch simple polar curves, for $0 \leqslant \theta < 2\pi$ or $-\pi < \theta \leqslant \pi$ or a subset of either of these intervals (detailed plotting of curves will not be required, but sketches will generally be expected to show significant features, such as symmetry, the form of the curve at the pole and least/greatest values of r); • recall the formula $\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, d\theta$ for the area of a sector, and use this formula in simple cases.

No changes to section 2

There are no changes proposed to the content of this section.

3. Summation of series	• use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums; • use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions; • recognise, by direct consideration of a sum to *n* terms, when a series is convergent, and find the sum to infinity in such cases.
4. Mathematical induction	• use the method of mathematical induction to establish a given result (questions set may involve divisibility tests and inequalities, for example); • recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. find the *n*th derivative of xe^x.

Section 3 has been moved to Section 1. Section 4 is moved to FP2 content
The content of Section 3 has been included in the new Section 1 content. The content of Section 4, if Structures 2 or 3 are used, would be moved to FP2.

Please see the FP2 content for details.

5. **Differentiation and integration**	• obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between y and x is defined implicitly or parametrically; • derive and use reduction formulae for the evaluation of definite integrals in simple cases; • use integration to find: ○ mean values and centroids of two- and three-dimensional figures (where equations are expressed in cartesian coordinates, including the use of a parameter), using strips, discs or shells as appropriate, ○ arc lengths (for curves with equations in cartesian coordinates, including the use of a parameter, or in polar coordinates), ○ surface areas of revolution about one of the axes (for curves with equations in cartesian coordinates, including the use of a parameter, but not for curves with equations in polar coordinates).

Moved to FP2 content

The content of this section would be included in FP2 if structure options 2 or 3 are used with some amendments to this content.

Please see the FP2 content for details.

6. Differential equations	• recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary function and a particular integral; • find the complementary function for a second order linear differential equation with constant coefficients; • recall the form of, and find, a particular integral for a second order linear differential equation in the cases where a polynomial or e^{bx} or $a \cos px + b \sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral; • use a substitution to reduce a given differential equation to a second order linear equation with constant coefficients; • use initial conditions to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.

First order differential equations has been inserted to give a more comprehensive approach to differential equations

3. Differential equations	• **find an integrating factor for a first order linear differential equation, and use an integrating factor to find the general solution;** • recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and recall that the general solution is the sum of the complementary function and a particular integral; • find the complementary function for a **first or** second order linear differential equation with constant coefficients; • recall the form of, and find, a particular integral for a **first or** second order linear differential equation in the cases where a polynomial or ae^{bx} or $a \cos px + b \sin px$ is a suitable form, and in other simple cases find the appropriate coefficient(s) given a suitable form of particular integral; • use a **given** substitution to reduce a differential equation to a **first or** second order linear equation with constant coefficients **or to a first order equation with separable variables**; • use initial conditions to find a particular solution to a differential equation, and interpret a solution in terms of a problem modelled by a differential equation.

7. **Complex numbers**	• understand de Moivre's theorem, for a positive integral exponent, in terms of the geometrical effect of multiplication of complex numbers;
	• prove de Moivre's theorem for a positive integral exponent;
	• use de Moivre's theorem for positive integral exponent to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle;
	• use de Moivre's theorem, for a positive or negative rational exponent:
	  ○ in expressing powers of sin θ and cos θ in terms of multiple angles,
	  ○ in the summation of series,
	  ○ in finding and using the nth roots of unity.

Moved to FP2 content
The content of this section would be included in FP2 if structure options 2 or 3 are used. Please see the FP2 content for details.

8. **Vectors**	• use the equation of a plane in any of the forms $ax + by + cz = d$ or $\mathbf{r}.\mathbf{n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, and convert equations of planes from one form to another as necessary in solving problems; • recall that the vector product $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\|\mathbf{a}\| \|\mathbf{b}\| \sin\theta\, \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector, or in component form as $(a_2b_3 - a_3b_2)\,\mathbf{i} + (a_3b_1 - a_1b_3)\,\mathbf{j} + (a_1b_2 - a_2b_1)\,\mathbf{k}$; • use equations of lines and planes, together with scalar and vector products where appropriate, to solve problems concerning distances, angles and intersections, including: ○ determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists, ○ finding the perpendicular distance from a point to a plane, ○ finding the angle between a line and a plane, and the angle between two planes, ○ finding an equation for the line of intersection of two planes, ○ calculating the shortest distance between two skew lines, ○ finding an equation for the common perpendicular to two skew lines.

Moved to FP2 content

The content of this section would be included in FP2 if structure options 2 or 3 are used. Please see the FP2 content for details.

If the vector content in A Level Mathematics 9709 is updated, any changes would be followed through in 9231.

9. Matrices and linear spaces	• recall and use the axioms of a linear (vector) space (restricted to spaces of finite dimension over the field of real numbers only); • understand the idea of linear independence, and determine whether a given set of vectors is dependent or independent; • understand the idea of the subspace spanned by a given set of vectors; • recall that a basis for a space is a linearly independent set of vectors that spans the space, and determine a basis in simple cases; • recall that the dimension of a space is the number of vectors in a basis; • understand the use of matrices to represent linear transformations from $\mathbb{R}^n \rightarrow \mathbb{R}^m$;

	• understand the terms 'column space', 'row space', 'range space' and 'null space', and determine the dimensions of, and bases for, these spaces in simple cases; • determine the rank of a square matrix, and use (without proof) the relation between the rank, the dimension of the null space and the order of the matrix; • use methods associated with matrices and linear spaces in the context of the solution of a set of linear equations; • evaluate the determinant of a square matrix and find the inverse of a non-singular matrix (2 × 2 and 3 × 3 matrices only), and recall that the columns (or rows) of a square matrix are independent if and only if the determinant is non-zero; • understand the terms 'eigenvalue' and 'eigenvector', as applied to square matrices; • find eigenvalues and eigenvectors of 2 × 2 and 3 × 3 matrices (restricted to cases where the eigenvalues are real and distinct); • express a matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a ... ix whose ... ssion,

Matrices

The proposed change to this content follows on the next page. The section would be renamed Matrices. The content would be reduced to improve the progression from matrix work studied previously. Please see the next page for details.

4. Matrices

- carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix;
- recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices and, for 2 × 2 and 3 × 3 matrices, evaluate determinants and find inverses of non-singular matrices (the notation det $\mathbf{M}$ for the determinant of matrix $\mathbf{M}$ is included);
- understand and use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$;
- understand the use of 2 × 2 matrices to represent certain geometric transformations in the x-y plane, and in particular:
  - recognise that the matrix product $\mathbf{AB}$ represents the transformation that results from the transformation represented by $\mathbf{B}$ followed by the transformation represented by $\mathbf{A}$,
  - recall how the area scale factor of a transformation is related to the determinant of the corresponding matrix,
  - find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required);
- formulate a problem involving the solution of 2 linear simultaneous equations in 2 unknowns, or 3 equations in 3 unknowns, as a problem involving the solution of a matrix equation, or vice versa;
- understand the cases that may arise concerning the consistency or inconsistency of 2 or 3 linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, solve consistent systems, and interpret geometrically in terms of lines or planes;
- understand the terms 'eigenvalue' and 'eigenvector', as applied to square matrices;
- find eigenvalues and eigenvectors of 2 × 2 and 3 × 3 matrices (restricted to cases where the eigenvalues are real and distinct);
- express a matrix in the form $\mathbf{QDQ}^{-1}$, where $\mathbf{D}$ is a diagonal matrix of eigenvalues and $\mathbf{Q}$ is a matrix whose columns are eigenvectors, and use this expression, e.g. in calculating powers of matrices.

This content has been amended to focus on the geometric and algebraic applications of matrices to represent transformations and solve equations. Eigenvalues and eigenvectors have been retained.

Proposed Further Pure 2 content (FP2)

Further Pure Mathematics 2 content

The remaining pure content from 9231 would be used to form a second unit, FP2. This unit would build upon upon the knowledge and understanding learnt in FP1. This unit would be compulsory for the A Level course and an option for the AS Level course. There would be no optional questions on this paper.

3. Summation of series	• use the standard results for $\sum r$, $\sum r^2$, $\sum r^3$ to find related sums; • use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions; • recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.
4. Mathematical induction	• use the method of mathematical induction to establish a given result (questions set may involve divisibility tests and inequalities, for example); • recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. find the nth derivative of xe^x.

The content of Section 3 has become part of the new Section 1 in FP1. Section 4 becomes the new Section 1 of FP2, with minor amendments to clarify the types of induction problems that could be set.

1. Mathematical induction	• use the method of mathematical induction to establish a given result (questions set will not be restricted to any particular topics and may involve sums of finite series, forms for the general term of a sequence, powers of matrices, divisibility tests and inequalities, for example); • recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. find the nth derivative of xe^x.

7. **Complex numbers**	• understand de Moivre's theorem, for a positive integral exponent, in terms of the geometrical effect of multiplication of complex numbers; • prove de Moivre's theorem for a positive integral exponent; • use de Moivre's theorem for positive integral exponent to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle; • use de Moivre's theorem, for a positive or negative rational exponent: ○ in expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles, ○ in the summation of series, ○ in finding and using the nth roots of unity.

2. Complex numbers	• understand de Moivre's theorem, for a positive and negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers; • prove de Moivre's theorem for a positive integer exponent; • use de Moivre's theorem to express trigonometrical ratios of multiple angles in terms of powers of trigonometrical ratios of the fundamental angle; • use de Moivre's theorem: ○ in expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles, ○ in the summation of series, ○ in finding and using the nth roots of unity.

The content of this section has been amended to include negative integer exponents and the geometrical effect when dividing complex numbers. This improves the coherence of the content.

5. Differentiation and integration	• obtain an expression for $\frac{d^2y}{dx^2}$ in cases where the relation between y and x is defined implicitly or parametrically; • derive and use reduction formulae for the evaluation of definite integrals in simple cases; • use integration to find: ○ mean values and centroids of two- and three-dimensional figures (where equations are expressed in cartesian coordinates, including the use of a parameter), using strips, discs or shells as appropriate, ○ arc lengths (for curves with equations in cartesian coordinates, including the use of a parameter, or in polar coordinates), ○ surface areas of revolution about one of the axes (for curves with equations in cartesian coordinates, including the use of a parameter, but not for curves with equations in polar coordinates).

The content of this section has been amended to include inverse trigonometrical functions, Maclaurin's Series, and mean values and content on centroids has been removed. The expectation that learners should be able to apply their knowledge of integration from A Level Mathematics to more complex problems is now clear.
This improves the breadth of the content and improves progression for further study.

3. Differentiation and integration	• derive and use the derivatives of inverse trigonometric functions; • integrate $\frac{1}{\sqrt{a^2 - x^2}}$ and $\frac{1}{a^2 + x^2}$ and use trigonometric substitutions to integrate associated functions (the substitution $t = \tan \frac{1}{2}x$ is included); • derive and use the first few terms of Maclaurin's series for a function (derivation of a general term is not required). • derive and use reduction formulae for the evaluation of definite integrals in simple cases; • use integration to find: ○ arc lengths (for curves with equations in cartesian coordinates, including the use of a parameter, or in polar coordinates), ○ surface areas of revolution about one of the axes (for curves with equations in cartesian coordinates, including the use of a parameter, but not for curves with equations in polar coordinates). The ability to apply integration techniques introduced in A level Mathematics to more complex problems is expected and will be required by the assessment.

New content on Hyperbolic Functions

A new section on Hyperbolic functions is proposed. This will improve the breadth of the content, allow more coherent connections to be made across the content and improve progression to further study.

4. Hyperbolic functions	• understand the definitions of the hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$, $\operatorname{sech} x$, $\operatorname{cosech} x$ and $\coth x$ in terms of the exponential function; • sketch the graphs of hyperbolic functions; • use identities involving hyperbolic functions; • differentiate and integrate hyperbolic functions; • understand and use the definitions of the inverse hyperbolic functions and derive and use their logarithmic forms and their derivatives; • recognise integrals of functions of the form $\frac{1}{\sqrt{x^2+a^2}}$ and $\frac{1}{\sqrt{x^2-a^2}}$, and integrate associated functions using hyperbolic substitutions.

8. Vectors	• use the equation of a plane in any of the forms $ax + by + cz = d$ or $\mathbf{r.n} = p$ or $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, and convert equations of planes from one form to another as necessary in solving problems; • recall that the vector product $\mathbf{a} \times \mathbf{b}$ of two vectors can be expressed either as $\|\mathbf{a}\|\,\|\mathbf{b}\| \sin\theta\,\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is a unit vector, or in component form as $(a_2b_3 - a_3b_2)\,\mathbf{i} + (a_3b_1 - a_1b_3)\,\mathbf{j} + (a_1b_2 - a_2b_1)\,\mathbf{k}$; • use equations of lines and planes, together with scalar and vector products where appropriate, to solve problems concerning distances, angles and intersections, including: ○ determining whether a line lies in a plane, is parallel to a plane or intersects a plane, and finding the point of intersection of a line and a plane when it exists, ○ finding the perpendicular distance from a point to a plane, ○ finding the angle between a line and a plane, and the angle between two planes, ○ finding an equation for the line of intersection of two planes, ○ calculating the shortest distance between two skew lines, ○ finding an equation for the common perpendicular to two skew lines.

Vectors

This section will be revised to remove explicit overlap with the vector content of A Level Mathematics, though harder problems involving lines and planes would be required. Use of the scalar triple product will be added to the content.

5. Vectors	• use the equations of a line in the form $\frac{x-a}{p} = \frac{y-b}{q} = \frac{z-c}{r}$ and the equation of a plane in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$, and convert equations of lines and planes between different forms, as required in solving problems; • calculate the shortest distance between two skew lines, • find an equation for the common perpendicular to two skew lines; • calculate the scalar triple product and use it to find the volume of a parallelepiped or a tetrahedron.

Further Applied Mathematics units

If structure 1 option is used
The applied mathematics content would continue to be assessed in a 3 hour paper which contributed 50% to the final grade.
The existing mechanics and statistics content would form the basis of Paper 2. Changes would be incorporated to follow through from any changes to the applied mathematics content in A Level Mathematics.
These potential changes are outlined in the sections below.
The optional question would be removed.

If structure 2 option is used
The applied mathematics content would be assessed in two papers, Further Mechanics and Further Statistics.
The existing mechanics and statistics content would form the basis of these papers. Changes would be incorporated to follow through from any changes to the applied mathematics content in A Level Mathematics.
These potential changes are outlined in the sections below.
The optional question would be removed.

If structure 3 option is used
The applied mathematics content would be assessed in two papers, Further Mechanics and Further Statistics.
Mechanics and statistics content would be introduced from Papers 5 and 7 of the A Level to form two new units.
Changes would be incorporated to follow through from any changes to the applied mathematics content in A Level Mathematics.
The optional question would be removed.
These potential changes are not outlined below. We would consult separately on these changes if this consultation supports Option 3.

Further Mechanics (FM)

Previous knowledge

The statement regarding previous knowledge would be clarified.

Unit FM: Further Mechanics (Paper 3)

Knowledge of the content of units M1 and M2 of Mathematics 9709 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of any relevant pure mathematics in unit FP1 (or preceding units) may also be required.

5.2 Paper 2

Knowledge of the syllabuses for Mechanics (units M1 and M2) and Probability and Statistics (units S1 and S2) in Mathematics 9709 is assumed. Candidates may need to apply such knowledge in answering questions; harder questions on those units may also be set.

Theme or topic	Curriculum objectives
	Candidates should be able to:
MECHANICS (Sections 1 to 5)	
1. Momentum and impulse	• recall and use the definition of linear momentum, and show understanding of its vector nature (in one dimension only); • recall Newton's experimental law and the definition of the coefficient of restitution, the property $0 \leqslant e \leqslant 1$, and the meaning of the terms 'perfectly elastic' ($e = 1$) and 'inelastic' ($e = 0$); • use conservation of linear momentum and/or Newton's experimental law to solve problems that may be modelled as the direct impact of two smooth spheres or the direct or oblique impact of a smooth sphere with a fixed surface; • recall and use the definition of the impulse of a constant force, and relate the impulse acting on a particle to the change of momentum of the particle (in one dimension only).

Momentum and Impulse
Most of this content will move into 9709. This would lead to more advanced topics being set in 9231. The content below would be inserted.

1. Momentum and impulse	• understand the vector nature of impulse and momentum for motion in two dimensions; • solve problems that may be modelled as the oblique impact ot two smooth spheres or the oblique impact of a smooth sphere with a fixed surface (the appropriate use of Newton's experimental law is included).

2. Circular motion	• recall and use the radial and transverse components of acceleration for a particle moving in a circle with variable speed; • solve problems which can be modelled by the motion of a particle in a vertical circle without loss of energy (including finding the tension in a string or a normal contact force, locating points at which these are zero, and conditions for complete circular motion).

No changes to section 2

There are no changes proposed to the content of this section.

3. Equilibrium of a rigid body under coplanar forces	• understand and use the result that the effect of gravity on a rigid body is equivalent to a single force acting at the centre of mass of the body, and identify the centre of mass by considerations of symmetry in suitable cases; • calculate the moment of a force about a point in 2 dimensional situations only (understanding of the vector nature of moments is not required); • recall that if a rigid body is in equilibrium under the action of coplanar forces then the vector sum of the forces is zero and the sum of the moments of the forces about any point is zero, and the converse of this; • use Newton's third law in situations involving the contact of rigid bodies in equilibrium; • solve problems involving the equilibrium of rigid bodies under the action of coplanar forces (problems set will not involve complicated trigonometry).

Section 3

The content in this section would be refined down to prevent explicit overlap with the content of A level Mathematics. There would be a slight change to the title of this section. These changes are shown below.

3. Equilibrium of rigid bodies under coplanar forces	• use Newton's third law in situations involving the contact of rigid bodies in equilibrium; • solve problems involving the equilibrium of rigid bodies under the action of coplanar forces (problems set will not involve complicated trigonometry).

4. Rotation of a rigid body	• understand and use the definition of the moment of inertia of a system of particles about a fixed axis as $\sum mr^2$ and the additive property of moment of inertia for a rigid body composed of several parts (the use of integration to find moments of inertia will not be required); • use the parallel and perpendicular axes theorems (proofs of these theorems will not be required); • recall and use the equation of angular motion $C = I\ddot{\theta}$ for the motion of a rigid body about a fixed axis (simple cases only, where the moment C arises from constant forces such as weights or the tension in a string wrapped around the circumference of a flywheel; knowledge of couples is not included and problems will not involve consideration or calculation of forces acting at the axis of rotation); • recall and use the formula $\frac{1}{2}I\omega^2$ for the kinetic energy of a rigid body rotating about a fixed axis; • use conservation of energy in solving problems concerning mechanical systems where rotation of a rigid body about a fixed axis is involved.

No changes to section 4

There are no changes proposed to the content of this section.

5. Simple harmonic motion	• recall a definition of SHM and understand the concepts of period and amplitude; • use standard SHM formulae in the course of solving problems; • set up the differential equation of motion in problems leading to SHM, recall and use appropriate forms of solution, and identify the period and amplitude of the motion; • recognise situations where an exact equation of motion may be approximated by an SHM equation, carry out necessary approximations (e.g. small angle approximations or binomial approximations) and appreciate the conditions necessary for such approximations to be useful.

No changes to section 5
There are no changes proposed to the content of this section.

Further Statistics (FS)

Previous knowledge

The statement regarding previous knowledge would be clarified.

Unit FS: Further Probability and Statistics (Paper 4)

Knowledge of the content of units S1 and S2 of Mathematics 9709 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of any relevant pure mathematics in unit FP1 (or preceding units) may also be required.

STATISTICS (Sections 6 to 9)	
6. Further work on distributions	• use the definition of the distribution function as a probability to deduce the form of a distribution function in simple cases, e.g. to find the distribution function for Y, where $Y = X^3$ and X has a given distribution; • understand conditions under which a geometric distribution or negative exponential distribution may be a suitable probability model; • recall and use the formula for the calculation of geometric or negative exponential probabilities; • recall and use the means and variances of a geometric distribution and negative exponential distribution.

Clarification of this section

The content in this section will be updated to reflect the changes that have been made in A Level Mathematics. This section will focus on continuous random variables, probability density functions and cumulative distribution functions. Geometric distributions will be included in A Level Mathematics.

6. Further work on continuous random variables	• use a probability density function which may be defined piecewise; • use the general result $E(g(X)) = \int f(x)g(x)\,dx$ where $f(x)$ is the probability density function of the continuous random variable X, and $g(X)$ is a function of X; • understand and use the relationship between the probability density function and the cumulative distribution function, and use either to evaluate probabilities or percentiles; • use cumulative distribution functions of related variables in simple cases, e.g. given the c.d.f. of a variable X, to find the c.d.f. and hence the p.d.f. of Y, where $Y = X^3$; • solve problems in which the negative exponential distribution is a suitable probability model, and recall and use the mean and variance of the negative exponential distribution.

7. **Inference using normal and *t*-distributions**	• formulate hypotheses and apply a hypothesis test concerning the population mean using a small sample drawn from a normal population of unknown variance, using a *t*-test; • calculate a pooled estimate of a population variance from two samples (calculations based on either raw or summarised data may be required); • formulate hypotheses concerning the difference of population means, and apply, as appropriate: ○ a 2-sample *t*-test, ○ a paired sample *t*-test, ○ a test using a normal distribution, (the ability to select the test appropriate to the circumstances of a problem is expected); • determine a confidence interval for a population mean, based on a small sample from a normal population with unknown variance, using a *t*-distribution; • determine a confidence interval for a difference of population means, using a *t*-distribution, or a normal distribution, as appropriate.

No changes to section 7, 8 or 9.
There are no changes proposed to the content of these sections.

Probability generating functions

This topic would be added to the content of Further Statistics.

8. Probability generating functions	• understand the concept of a probability generating function and construct and use the probability generating function for given distributions (including the discrete uniform, binomial, geometric and Poisson distributions); • use formulae for the mean and variance of a discrete random variable in terms of its probability generating function, and use these formulae to calculate the mean and variance of probability distributions; • use the result that the probability generating function of the sum of independent random variables is the product of the probability generating functions of those random variables.

7. Mathematical notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

1 Set notation

Mathematical notation

The notation could be refined down to remove any notation that is not relevant for A Level Further Mathematics

$\in$	is an eler
$\notin$	is not an
$\{x_1, x_2, \ldots\}$	the set w
$\{x : \ldots\}$	the set o
$\mathrm{n}(A)$	the numb
$\varnothing$	the empty set
$\mathscr{E}$	the universal set
A'	the complement of the set A
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
$\mathbb{Z}_n$	the set of integers modulo n, $\{0, 1, 2, \ldots, n-1\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
$\mathbb{Q}^+$	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
$\mathbb{Q}_0^+$	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{R}^+$	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
$\mathbb{R}_0^+$	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
$\mathbb{C}$	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B, i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
$\subseteq$	is a subset of
$\subset$	is a proper subset of
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y\,R\,x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x, in the context of some equivalence relation