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### Abbreviations

- **awrt**: answers which round to
- **cao**: correct answer only
- **dep**: dependent
- **FT**: follow through after error
- **isw**: ignore subsequent working
- **oe**: or equivalent
- **rot**: rounded or truncated
- **SC**: Special Case
- **soi**: seen or implied
- **www**: without or wrong working

### Question 1

(i) \[180^\circ \text{ or } \pi \text{ radians or 3.14 radians (or better)}\]  

(ii) 2

(iii) (a)  

(b) ![Graph](image)

(iv) 3

### Question 2

(i) \[
\tan \theta = \frac{8 + 5\sqrt{2}}{4 + 3\sqrt{2}} \left(4 - 3\sqrt{2}\right)\\
= \frac{32 - 24\sqrt{2} + 20\sqrt{2} - 30}{16 - 18}\\
= 1 + 2\sqrt{2} \text{ (cao)}
\]

### Mark Scheme

<table>
<thead>
<tr>
<th>Question</th>
<th>Mark</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (i)</td>
<td>B1</td>
<td>180° or π radians or 3.14 radians (or better)</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>y = \sin 2x \ all correct</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>for either [\uparrow\downarrow\uparrow] starting at their highest value and ending at their lowest value</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>Or a curve with highest value at (y = 3) and lowest value at (y = -1)</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>completely correct graph</td>
</tr>
<tr>
<td>2 (i)</td>
<td>M1</td>
<td>attempt to obtain (\tan \theta) and rationalise. Must be convinced that no calculators are being used</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td></td>
</tr>
</tbody>
</table>
(ii) \[
\sec^2 \theta = 1 + \tan^2 \theta \\
= 1 + (1 + 2\sqrt{2})^2 \\
= 1 + 1 - 4\sqrt{2} + 8 \\
= 10 - 4\sqrt{2}
\]

Alternative solution:
\[
AC^2 = (4 + 3\sqrt{2})^2 + (8 + 5\sqrt{2})^2 \\
= 148 + 104\sqrt{2}
\]
\[
\sec^2 \theta = \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})^2} \\
= \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})} \times \frac{34 - 24\sqrt{2}}{34 - 24\sqrt{2}} \\
= 10 - 4\sqrt{2}
\]

| 3 (i) | \[64 + 192x^2 + 240x^4 + 160x^6\] | B3,1,0 | \(-1\) each error |
| (ii) | \[(64 + 192x^2 + 240x^4)\left(1 - \frac{6}{x^2} + \frac{9}{x^4}\right)\] | B1 | expansion of \(\left(1 - \frac{3}{x^2}\right)^2\) |
| | Terms needed \(64 - (192 \times 6) + (240 \times 9)\) | M1 | attempt to obtain 2 or 3 terms using their (i) |
| | = 1072 | A1 | |

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4 (a) \[ X^2 = \begin{pmatrix} 4 & -4k \\ 2k & -4k \end{pmatrix} \]

(b) Use of \( \mathbf{A}^{-1} = \mathbf{I} \)

\[
\begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 6 & -6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}
\]

Any 2 equations will give \( a = 2, b = 4 \)

**Alternative method 1:**

\[
\frac{1}{5a-b} \begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix} = \begin{pmatrix} 5 & -1 \\ 6 & -6 \end{pmatrix}
\]

Compare any 2 terms to give \( a = 2, b = 4 \)

**Alternative method 2:**

Inverse of \( \begin{pmatrix} 5 & -1 \\ 6 & -4 \end{pmatrix} \) = \( \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix} \)

5

\[ 3x - 1 = x(3x - 1) + x^2 - 4 \text{ or } \]

\[ y = \left( \frac{y+1}{3} \right)y + \left( \frac{y+1}{3} \right)^2 - 4 \]

\[ 4x^2 - 4x - 3 = 0 \text{ or } 4y^2 - 4y - 35 = 0 \]

\[ (2x - 3)(2x + 1) = 0 \text{ or } (2y - 7)(2y + 5) = 0 \]

leading to \( x = \frac{3}{2} \), \( x = -\frac{1}{2} \) and

\( y = \frac{7}{2} \), \( y = -\frac{5}{2} \)

Midpoint \( \left( \frac{1}{2}, \frac{1}{2} \right) \)

Perpendicular gradient = \( -\frac{1}{3} \)

Perp bisector: \( y - \frac{1}{2} = -\frac{1}{3} \left( x - \frac{1}{2} \right) \)

\( (3y + x - 2 = 0) \)
6 (i) \[ f\left(\frac{1}{2}\right) = a - \frac{15}{4} + b - 2 = 0 \]
leading to \( a + 4b = 46 \)
\[ f(1) = a - 15 + b - 2 = 5 \]
leading to \( a + b = 22 \)
giving \( b = 8 \) (AG), \( a = 14 \)

(ii) \( (2x - 1)(7x^2 - 4x + 2) \)

(iii) \( 7x^2 - 4x + 2 = 0 \) has no real solutions as \( b^2 < 4ac \)
\[ 16 < 56 \]

7 (i) \[ \frac{dy}{dx} = \frac{(x-1) \cdot \frac{8x}{4x^2 + 2} - \ln(4x^2 + 3)}{(x-1)^2} \]

When \( x = 0 \), \( y = -\ln 3 \) \( \text{oe} \)
\[ \frac{dy}{dx} = -\ln 3 \] so gradient of normal is \( \frac{1}{\ln 3} \)
(allow numerical equivalent)

normal equation \( y + \ln 3 = \frac{1}{\ln 3} x \)
or \( y = 0.910x - 1.10 \), or \( y = \frac{10}{11} x - \frac{11}{10} \) \( \text{cao} \)
(Allow \( y = 0.91x - 1.1 \))

(ii) when \( x = 0 \), \( y = -\ln 3 \)
when \( y = 0 \), \( x = (\ln 3)^3 \)
Area = \( \pm 0.66 \) or \( \pm 0.67 \) or awrt these
or \( \frac{1}{2} (\ln 3)^3 \)
<p>| | | | |</p>
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</table>
|8 | (i) | Range for f: \( y \geq 3 \)  
Range for g: \( y \geq 9 \) | B1  
B1 |
|   | (ii) |  
\[ x = -2 + \sqrt{y - 5} \] 
\[ g^{-1}(x) = -2 + \sqrt{x - 5} \]  
Domain of \( g^{-1} \): \( x \geq 9 \) | M1  
A1  
B1 |
|   |   |   | attempt to obtain the inverse function  
A1 Must be correct form  
B1 for domain |
|   | Alternative method: |  
\[ y^2 + 4y + 9 - x = 0 \]  
\[ y = \frac{-4 + \sqrt{16 - 4(9-x)}}{2} \] | M1  
A1  
A1 |
|   |   |   | attempt to use quadratic formula and find inverse  
A1 must have + not ± |
|   | (iii) | Need \( g(3e^{2x}) \)  
\[ (3e^{2x} + 2)^2 + 5 = 41 \]  
or \[ 9e^{4x} + 12e^{2x} - 32 = 0 \]  
\[ (3e^{2x} - 4)(3e^{2x} + 8) = 0 \]  
leading to \( 3e^{2x} + 2 = \pm 6 \) so \( x = \frac{1}{2} \ln \frac{4}{3} \)  
or \( e^{2x} = \frac{4}{3} \) so \( x = \frac{1}{2} \ln \frac{4}{3} \) | M1  
DM1  
M1  
A1  
A1 |
|   |   |   | M1 dealing with the exponential correctly in order to reach a solution for \( x \)  
A1 Allow equivalent logarithmic forms |
|   | Alternative method: | Using \( f(x) = g^{-1}(41) \), \( g^{-1}(41) = 4 \)  
leading to \( 3e^{2x} = 4 \), so \( x = \frac{1}{2} \ln \frac{4}{3} \) | M1  
DM1  
M1  
A1  
A1 |
|   |   |   | M1 dealing with \( g^{-1}(41) \) to obtain an equation in terms of \( e^{2x} \)  
M1 dealing with the exponential correctly in order to reach a solution for \( x \)  
Allow equivalent logarithmic forms |
|   | (iv) | \( g'(x) = 6e^{2x} \)  
\( g' (\ln 4) = 96 \) | B1  
B1 |
|   |   |   | B1 for each |
9 (i) \[
\frac{dy}{dx} = 3x^2 - 10x + 3
\]
When \(x = 0\), for curve \(\frac{dy}{dx} = 3\),
gradient of line also 3 so line is a tangent.

Alternate method:
\[
3x + 10 = x^3 - 5x^2 + 3x + 10
\]
leading to \(x^2 = 0\), so tangent at \(x = 0\)

(ii) When \(\frac{dy}{dx} = 0\), \((3x - 1)(x - 3) = 0\)
\[
x = \frac{1}{3}, \ x = 3
\]

(iii) Area = \[
\frac{1}{2} (10 + 19) 3 - \int_{0}^{3} (x^3 - 5x^2 + 3x + 10) \ dx
\]
\[
= \frac{87}{2} \left[ \frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 10x \right]_0
\]
\[
= \frac{87}{2} \left( \frac{81}{4} - 45 + \frac{27}{2} + 30 \right)
\]
\[
= 24.7 \text{ or } 24.8
\]

Alternative method:
Area = \[
\int_{0}^{3} (3x + 10) - (x^3 - 5x^2 + 3x + 10) \ dx
\]
\[
= \int_{0}^{3} -x^3 + 5x^2 \ dx
\]
\[
= \left[ \frac{x^4}{4} + \frac{5x^3}{3} \right]_0^{99} = \frac{99}{4}
\]

10 (a) \[
\sin^2 x = \frac{1}{4}
\]
\[
\sin x = (\pm) \frac{1}{2}
\]
\[
x = 30^\circ, 150^\circ, 210^\circ, 330^\circ
\]

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### (b)

\[
\left( \sec^2 3y - 1 \right) - 2 \sec 3y = -2 = 0
\]

\[
\sec^2 3y - 2 \sec 3y - 3 = 0
\]

\[
(\sec 3y + 1)(\sec 3y - 3) = 0
\]

leading to \( \cos 3y = -1, \cos 3y = \frac{1}{3} \)

\( 3y = 180^\circ, 540^\circ \quad 3y = 70.5^\circ, 289.5^\circ, 430.5^\circ \)

\( y = 60^\circ, 180^\circ, 23.5^\circ, 96.5^\circ, 143.5^\circ \)

**Alternative 1:**

\[
\sec^2 3y - 2 \sec 3y - 3 = 0
\]

leading to \( 3 \cos^2 3y + 2 \cos 3y - 1 \)

\[
(3 \cos 3y - 1)(\cos 3y + 1) = 0
\]

**Alternative 2:**

\[
\frac{\sin^2 y}{\cos^2 y} - 2 \frac{\sin y}{\cos y} - 2 = 0
\]

\[
(1 - \cos^2 x) - 2 \cos x - 2 \cos^2 x = 0
\]

### (c)

\[
\frac{\pi}{3} = \frac{4\pi}{3} = \frac{2\pi}{3} = \frac{5\pi}{3}
\]

\[
z = \frac{2\pi}{3}, \frac{5\pi}{3} \quad \text{or} \quad 2.09 \quad 2.1, 5.24
\]

| M1 | use of the correct identity |
| M1 | attempt to obtain a 3 term quadratic equation in sec 3y and attempt to solve |
| M1 | dealing with sec and 3y correctly |

**A1, A1**

A1 for a correct pair, A1 for a second correct pair, A1 for correct 5th solution and no other within the range

| M1 | use of the correct identity |
| M1 | attempt to obtain a quadratic equation in cos 3y and attempt to solve |
| M1 | dealing with 3y correctly |
| A marks as above |

**M1**

use of the correct identity, 
\[
\tan y = \frac{\sin y}{\cos y} \quad \text{and} \quad \sec y = \frac{1}{\cos y}
\]

as before

| M1 | correct order of operations |

**A1, A1**

A1 for a correct solution
A1 for a second correct solution and no other within the range