READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in. Write in dark blue or black pen. You may use an HB pencil for any diagrams or graphs. Do not use staples, paper clips, glue or correction fluid. DO NOT WRITE IN ANY BARCODES.

Answer all the questions. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an electronic calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together. The number of marks is given in brackets [ ] at the end of each question or part question. The total number of marks for this paper is 80.
1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)! r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\cosec^2 A = 1 + \cot^2 A$$

Formulae for $\Delta ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$
1 Without using a calculator, express \( \frac{(2 + \sqrt{5})^2}{\sqrt{5} - 1} \) in the form \( a + b\sqrt{5} \), where \( a \) and \( b \) are constants to be found. \[4\]

2 Find the values of \( k \) for which the line \( y + kx - 2 = 0 \) is a tangent to the curve \( y = 2x^2 - 9x + 4 \). \[5\]
3 (i) Given that $x + 1$ is a factor of $3x^3 - 14x^2 - 7x + d$, show that $d = 10$. [1]

(ii) Show that $3x^3 - 14x^2 - 7x + 10$ can be written in the form $(x + 1)(ax^2 + bx + c)$, where $a$, $b$ and $c$ are constants to be found. [2]

(iii) Hence solve the equation $3x^3 - 14x^2 - 7x + 10 = 0$. [2]
4  (i) Express \(12x^2 - 6x + 5\) in the form \(p(x - q)^2 + r\), where \(p, q\) and \(r\) are constants to be found. [3]

(ii) Hence find the greatest value of \(\frac{1}{12x^2 - 6x + 5}\) and state the value of \(x\) at which this occurs. [2]
5 (i) Find and simplify the first three terms of the expansion, in ascending powers of \(x\), of \((1 - 4x)^5\). [2]

(ii) The first three terms in the expansion of \(\left(1 - 4x\right)^5 \left(1 + ax + bx^2\right)\) are \(1 - 23x + 222x^2\). Find the value of each of the constants \(a\) and \(b\). [4]
6  (a)  (i) State the value of \( u \) for which \( \lg u = 0 \). [1]

(ii) Hence solve \( \lg |2x + 3| = 0 \). [2]

(b) Express \( 2 \log_3 15 - (\log_a 5)(\log_3 a) \), where \( a > 1 \), as a single logarithm to base 3. [4]
Differentiate with respect to $x$

(i) $x^4 e^{3x}$, [2]

(ii) $\ln(2 + \cos x)$, [2]

(iii) $\frac{\sin x}{1 + \sqrt{x}}$. [3]
8 The line $y = x - 5$ meets the curve $x^2 + y^2 + 2x - 35 = 0$ at the points $A$ and $B$. Find the exact length of $AB$. [6]
A curve is such that \( \frac{dy}{dx} = (2x + 1)^{\frac{1}{2}} \). The curve passes through the point (4, 10).

(i) Find the equation of the curve. \[4\]

(ii) Find \( \int y \, dx \) and hence evaluate \( \int_{0}^{1.5} y \, dx \). \[5\]
Two variables $x$ and $y$ are connected by the relationship $y = Ab^x$, where $A$ and $b$ are constants.

(i) Transform the relationship $y = Ab^x$ into a straight line form.

(ii) Use the graph to determine the value of $A$ and the value of $b$, giving each to 1 significant figure.

(iii) Find $x$ when $y = 220$. 
The functions $f$ and $g$ are defined, for real values of $x$ greater than 2, by

$$f(x) = 2^x - 1,$$
$$g(x) = x(x + 1).$$

(i) State the range of $f$.  

(ii) Find an expression for $f^{-1}(x)$, stating its domain and range.
(iii) Find an expression for \(gf(x)\) and explain why the equation \(gf(x) = 0\) has no solutions. [4]
12 A curve has equation \( y = x^3 - 9x^2 + 24x \).

(i) Find the set of values of \( x \) for which \( \frac{dy}{dx} \geq 0 \). [4]

The normal to the curve at the point on the curve where \( x = 3 \) cuts the \( y \)-axis at the point \( P \).

(ii) Find the equation of the normal and the coordinates of \( P \). [5]