CAMBRIDGE
International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/02**

Paper 2 Further Pure Mathematics 2 **For examination from 2027**

SPECIMEN PAPER **2 hours**

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You must answer on the printed answer booklet.

You will need: Printed answer booklet
List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Write your answers on the printed answer booklet. Follow the instructions on the front cover of the answer booklet.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **4** pages.

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[Turn over

v1

1 Find the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 4x = 7 - 2t^2.$$ [6]

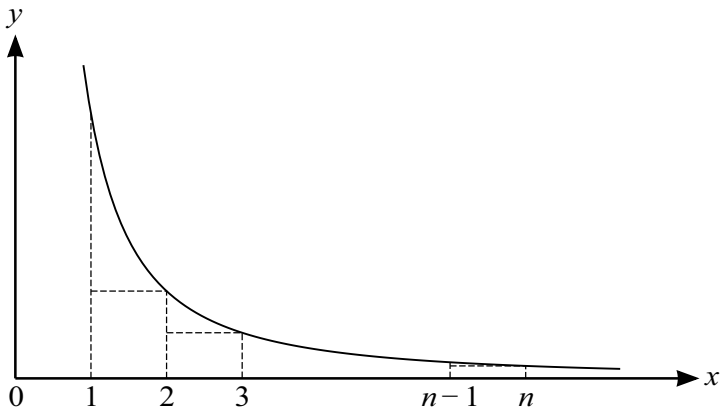
2 Find the exact value of $\displaystyle\int_0^1 \frac{1}{\sqrt{3 + 4x - 4x^2}}\mathrm{d}x$. [6]

3 Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{\sin x}{x}$$

for which $y = 0$ when $x = \frac{1}{2}\pi$. Give your answer in the form $y = \mathrm{f}(x)$. [8]

4



The diagram shows the curve with equation $y = \dfrac{1}{x^2}$ for $x > 0$, together with a set of $(n - 1)$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{r^2} < \frac{2n-1}{n}.$$ [5]

(b) Use a similar method to find, in terms of n, a lower bound for $\displaystyle\sum_{r=1}^{n} \frac{1}{r^2}$. [3]

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5 The curve C has parametric equations

$$x = e^t - 4t + 3, \qquad y = 8e^{\frac{1}{2}t}, \qquad \text{for } 0 \leqslant t \leqslant 2.$$

(a) Find, in terms of e, the length of C. [5]

(b) Find, in terms of π and e, the area of the surface generated when C is rotated through 2π radians about the x-axis. [5]

6 **(a)** Using de Moivre's theorem, show that

$$\tan 5\theta = \frac{5\tan\theta - 10\tan^3\theta + \tan^5\theta}{1 - 10\tan^2\theta + 5\tan^4\theta}.$$ [5]

(b) Hence show that the equation $x^2 - 10x + 5 = 0$ has roots $\tan^2\left(\frac{1}{5}\pi\right)$ and $\tan^2\left(\frac{2}{5}\pi\right)$. [5]

7 **(a)** Starting from the definition of tanh in terms of exponentials, prove that $\tanh^{-1}x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$. [3]

(b) Given that $y = \tanh^{-1}\left(\frac{1-x}{2+x}\right)$, show that $(2x + 1)\frac{dy}{dx} + 1 = 0$. [4]

(c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\frac{1-x}{2+x}\right)$ in the form

$$a\ln 3 + bx + cx^2,$$

where a, b and c are constants to be determined. [5]

Question 8 is on the back page.

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8 **(a)** **(i)** Find the set of values of a for which the system of equations

$$\begin{aligned} x - 2y - 2z + 7 &= 0, \\ 2x + (a-9)y - 10z + 11 &= 0, \\ 3x - 6y + 2az + 29 &= 0, \end{aligned}$$

has a unique solution. [4]

(ii) Given that $a = -3$, show that the system of equations in **8(a)(i)** has no solution. Interpret this situation geometrically. [3]

(b) The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}.$$

(i) Find the eigenvalues of $\mathbf{A}$. [4]

(ii) Use the characteristic equation of $\mathbf{A}$ to find $\mathbf{A}^{-1}$. [4]

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