CAMBRIDGE
International Education

Cambridge International AS & A Level

FURTHER MATHEMATICS **9231/01**

Paper 1 Further Pure Mathematics 1 **For examination from 2027**

MARK SCHEME

Maximum Mark: 75

Specimen

This document has **12** pages.

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General Marking Principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: • the specific **content** defined in the mark scheme • the specific **skills** defined in the mark scheme or in the level descriptions • the **standard of response** required as exemplified by the standardisation scripts.
2	Examiners must award **whole marks** (not half marks, or other fractions).
3	Examiners must award marks **positively**. This means that: • marks are not deducted for errors or omissions • marks are awarded for correct/valid answers as defined in the mark scheme and also for **other valid answers** which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate • the quality of spelling, punctuation and grammar are judged **only** when the mark scheme indicates that these features are **specifically assessed** in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must **consistently** apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the **full range of marks** defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and **must not** award marks with grade thresholds or grade descriptions in mind.

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Mathematics-Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

Marks are of the following three types.

M	Method mark, given for a valid method applied to the problem. Method marks can still be given even if there are numerical errors, algebraic slips or errors in units. However the method must be applied to the specific problem, e.g. by substituting the relevant quantities into a formula. Correct use of a formula without the formula being quoted earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, given for an accurate answer or accurate intermediate step following a correct method. Accuracy marks cannot be given unless the relevant method mark has also been given.
B	Mark for a correct statement or step.
DM or DB	M marks and B marks are generally independent of each other. The notation DM or DB means a particular M or B mark is dependent on an earlier M or B mark (indicated by *). When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT below).
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures (sf) or would be correct to 3 sf if rounded (1 decimal place (dp) in the case of an angle in degrees). As stated above, an A or B mark is not given if a correct numerical answer is obtained from incorrect working.
- Common alternative solutions are shown in the Answer column as: '**EITHER** Solution 1 **OR** Solution 2 **OR** Solution 3 …'. Round brackets appear in the Partial Marks column around the marks for each alternative solution.
- The total number of marks available for each question is shown at the bottom of the Marks column in bold type.
- Square brackets [ ] around text show extra information not needed for the mark to be awarded.

The following abbreviations may be used in a mark scheme.

AG	Answer given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
CAO	Correct answer only (emphasising that no 'follow through' from an error is allowed).
CWO	Correct working only
FT	Follow through after error (see Mark Scheme Notes for further details).
ISW	Ignore subsequent working
OE	Or equivalent form
SC	Special case
SOI	Seen or implied

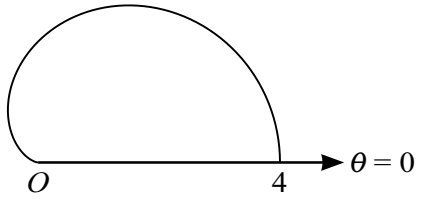
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Question	Answer	Marks	Partial Marks	Guidance
1(a)	$f(r-1)-f(r)=\frac{1}{r(r+1)}-\frac{1}{(r+1)(r+2)}$	1	**M1**	
	$=\frac{r+2-r}{r(r+1)(r+2)}=\frac{2}{r(r+1)(r+2)}$	1	**A1**	AG
		2		
1(b)	$\sum_{r=1}^{n}\frac{1}{r(r+1)(r+2)}=\frac{1}{2}\left(\frac{1}{1\times 2}-\frac{1}{2\times 3}\right)+\ldots+\frac{1}{2}\left(\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right)$	2	**M1A1**	Sets up method of differences
	$=\frac{1}{4}-\frac{1}{2}\left(\frac{1}{(n+1)(n+2)}\right)$	1	**B1**	OE Shows cancellation
		3		
1(c)	$\sum_{r=1}^{\infty}\frac{1}{r(r+1)(r+2)}=\frac{1}{4}$	**1**	**B1FT**	Follow through from *their* part **1(b)**.

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Question	Answer	Marks	Partial Marks	Guidance
2	**EITHER** Solution 1 $\phi(1) = 5 \times 5 - 1 = 24$ which is divisible by 8; P_1 true	1	**(B1**	
	Assume P_k true for some positive integer k: so $\phi(k) = 8l$	1	**B1**	
	$\phi(k + 1) - \phi(k)$	1	**M1**	
	$= 5^{k+1}(4k + 5) - 1 - 5^k(4k + 1) + 1$	1	**A1**	
	$= 5^k(16k + 24) = 8m$	1	**A1**	
	$\phi(k + 1) = 8(l + m)$	1	**A1**	
	Hence, by Principle of Mathematical Induction (PMI), true for all positive integers	1	**A1)**	
	OR Solution 2 $\phi(1) = 5 \times 5 - 1 = 24$ which is divisible by 8; P_1 true	1	**(B1**	
	Assume P_k true for some positive integer k: so $\phi(k) = 8l$	1	**B1**	
	$\phi(k + 1) = 5^{k+1}(4k + 5) - 1$ $= 5 \times (4k \times 5^k) + 25 \times 5^k - 1$ $= 5(8l - 5^k + 1) + 25 \times 5^k - 1$	2	**M1A1**	
	$= 40l + 20 \times 5^k + 4$	1	**A1**	
	$= 40l + 4(5^{k+1} + 1)$ $= 40l + 4(2m)$ since $(5^{k+1} + 1)$ is even so is divisible by 8	1	**A1**	Or equivalent argument
	Hence, by PMI, true for all positive integers	1	**A1)**	
	Available marks	**7**		

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Question	Answer	Marks	Partial Marks	Guidance
3(a)		1	**B1**	Shows (4, 0) and (0, π) lie on C
		1	**B1**	Section $\frac{\pi}{2} < \theta < \pi$ correct
		1	**B1**	Correct shape with curve perpendicular to initial line at (4, 0).
		3		
3(b)	$\frac{1}{2}\int_0^{\pi}(4 + 8\cos\theta + 4\cos^2\theta)\,\mathrm{d}\theta$	1	**M1**	Uses correct formula
	$= \int_0^{\pi}(3 + 4\cos\theta + \cos 2\theta)\,\mathrm{d}\theta$	1	**M1**	Uses double angle formula
	$\left[3\theta + 4\sin\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi} = 3\pi$	2	**A1A1**	
		4		
3(c)	$\sqrt{x^2 + y^2} = 2 + 2\,\frac{x}{\sqrt{x^2 + y^2}}$	2	**M1A1**	
	$2\sqrt{x^2 + y^2} = x^2 + y^2 - 2x \Rightarrow 4(x^2 + y^2) = (x^2 + y^2 - 2x)^2$	1	**A1**	AG Intermediate step required
		3		

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Question	Answer	Marks	Partial Marks	Guidance
4(a)	**EITHER** Solution 1 Let S_n denote $\alpha^n + \beta^n + \gamma^n$ $S_2 = S_1^{\,2} - 2\sum\alpha\beta = 1^2 - 2 \times (-1) = 3$	2	**(M1A1**	
	$S_3 = S_2 + S_1 + 15 = 3 + 1 + 15 = 19$	2	**M1A1)**	
	OR Solution 2 Use $\Sigma\alpha^3 = (\Sigma\alpha)^3 - 3(\Sigma\alpha)(\Sigma\alpha\beta) + 3\alpha\beta\gamma$	1	**(M1**	
	$\Sigma\alpha = 1$, $\Sigma\alpha\beta = -1$, $\alpha\beta\gamma = 5$	1	**B1**	
	$\Sigma\alpha^3 = 1 - 3 \times 1 \times -1 + 3 \times 5$	1	**M1**	
	$= 19$	1	**A1)**	
	Available marks	**4**		
4(b)	$S_4 = S_3 + S_2 + 5S_1 = 19 + 3 + 5 = 27$	**2**	**M1A1**	
4(c)	$x = z + 1 \Rightarrow z = x - 1$	1	**B1**	
	Required eqn: $(x-1)^3 - (x-1)^2 - (x-1) - 5 = 0$	1	**M1**	
	So, $x^3 - 4x^2 + 4x - 6 = 0$	1	**A1**	
		3		

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Question	Answer	Marks	Partial Marks	Guidance
5(a)	$-20 + 3k = 0$	1	**M1**	
	$k = \frac{20}{3}$	1	**A1**	
		2		
5(b)	Consider $\begin{pmatrix} 5 & 6 \\ -3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5x+6y \\ -3x-4y \end{pmatrix} \left(= \begin{pmatrix} X \\ Y \end{pmatrix}\right)$	1	**M1**	
	Line through O, so $y = mx$	1	**M1**	
	Invariant line, so $Y = mX$, $\Rightarrow -3x - 4mx = m(5x + 6mx)$	2	**M1A1**	
	$6m^2 + 9m + 3 = 0$ $m = -1, -\frac{1}{2}$	1	**A1**	
	Invariant lines $y = -x$, $2y + x = 0$	1	**A1**	
		6		
5(c)(i)	det **A** $= -20 + 18 = -2$	1	**M1**	
	Area $= 2 \times 10 = 20\,\text{cm}^2$	1	**A1**	
		2		
5(c)(ii)	$\mathbf{A}^{-1} = \frac{1}{-2}\begin{pmatrix} -4 & -6 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ -\frac{3}{2} & -\frac{5}{2} \end{pmatrix}$	**2**	**M1A1**	

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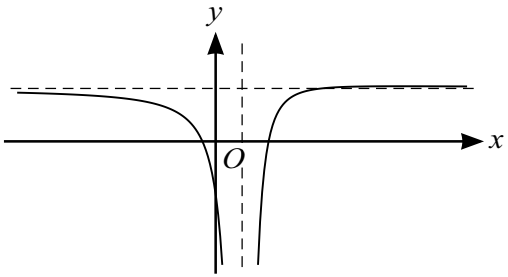
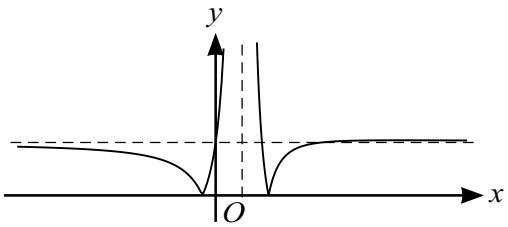
Question	Answer	Marks	Partial Marks	Guidance
6(a)	$\mathbf{n} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -1 & 1 \\ 0 & 1 & m-1 \end{vmatrix} = -m\mathbf{i} - 4(m-1)\mathbf{j} + 4\mathbf{k}$	2	**M1A1**	Finds direction of common perpendicular
	$\dfrac{\left\lvert \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} -m \\ 4-4m \\ 4 \end{pmatrix} \right\rvert}{\sqrt{(m^2 + 16(1-2m+m^2) + 16)}} = 3$	2	**M1A1**	Uses result for shortest distance between lines
	$19m^2 - 40m + 4 = 0$	2	**M1A1**	
	$(19m - 2)(m - 2) = 0 \quad m = 2$, since m is an integer	1	**A1**	AG Dependent on method seen for solving quadratic equation
		7		
6(b)	$\overrightarrow{CA} = \begin{pmatrix} 1 \\ 0 \\ -4 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ or $\overrightarrow{AD} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$	1	**B1**	Finds relevant vectors
	$\dfrac{1}{\sqrt{17}} \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 1 \\ 1 & 0 & -4 \end{vmatrix} = \dfrac{1}{\sqrt{17}} \begin{pmatrix} -4 \\ 1 \\ -1 \end{pmatrix}$	1	**M1**	Uses cross-product The unit vector parallel to $\overrightarrow{CA}$ is required
	$\dfrac{1}{\sqrt{17}} \sqrt{4^2 + 1^2 + 1^2} = \sqrt{\dfrac{18}{17}} (= 1.03)$	1	**A1**	
		3		

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Question	Answer	Marks	Partial Marks	Guidance
6(c)	$\overrightarrow{BC} = 3\mathbf{i} - \mathbf{j} + 5\mathbf{k}$	1	**B1**	Finds second vector in *BCD*
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 5 \\ 0 & 1 & 1 \end{vmatrix} = -6\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$	1	**M1**	Finds normal vector to *BCD*
	$\cos\theta = \dfrac{(4\mathbf{i} - \mathbf{j} + \mathbf{k}) \cdot (2\mathbf{i} + \mathbf{j} - \mathbf{k})}{\sqrt{16+1+1}\sqrt{4+1+1}} = \dfrac{6}{\sqrt{18}\sqrt{6}} = \dfrac{1}{\sqrt{3}}$	1	**M1**	Finds angle between normal vectors
	Angle between planes $= \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$	1	**A1**	AG
		4		

Question	Answer	Marks	Partial Marks	Guidance
7(a)	Vertical: $x = 1$ horizontal: $y = 2$	**2**	**B1B1**	
7(b)	$yx^2 - 2yx + y = 2x^2 - 3x - 2$	1	**M1**	
	$(y-2)x^2 - (2y-3)x + (y+2) = 0$	1	**A1**	
	For real x, $(2y-3)^2 - 4(y-2)(y+2) \geqslant 0$	1	**M1**	
	$12y \leqslant 25$, so $y \leqslant \frac{25}{12}$	1	**A1**	AG
		4		
7(c)	At turning points, $(x^2 - 2x + 1)(4x - 3) - (2x^2 - 3x - 2)(2x - 2) = 0$	1	**M1**	
	$x^2 - 8x + 7 = 0$ $x = 7$ (since $x = 1$ is an asymptote)	1	**M1**	
	Stationary point $\left(7, \frac{25}{12}\right)$	1	**A1**	
		3		

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Question	Answer	Marks	Partial Marks	Guidance
7(d)		1	**B1**	Axes and asymptotes
		1	**B1**	(–0.5, 0) (2, 0) (0, –2) (4, 2)
		1	**B1**	Left hand branch
		1	**B1**	Right hand branch
		4		
7(e)	 Correct sketch, FT from **(d)**	1	**B1FT**	FT from sketch in **(d)**
	Finds 0, $\frac{7}{4}$ and 4 as critical points	1	**M1**	Finds critical points
	$x < 0$	1	**B1**	
	$\frac{7}{4} < x < 4$	1	**A1**	
		4		

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