

Cambridge IGCSE[™]

CANDIDATE NAME							
CENTRE NUMBER		CANDIDATE NUMBER					
CAMBRIDGE INTERNATIONAL MATHEMATICS 0607/00							
Paper 6 Investi	stigation and Modelling (Extended) For examination fro						

You must answer on the question paper.

1 hour 30 minutes

No additional materials are needed.

INSTRUCTIONS

SPECIMEN PAPER

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages. Any blank pages are indicated.

Section A INVESTIGATION PICK'S FORMULA

You are advised to spend no more than 45 minutes on this section.

This investigation is about working out the area of a polygon.

In 1899 the Austrian mathematician Georg Pick found a method to work out the area, A, of any polygon that has its vertices on a square grid.

His method used the number of dots, p, on the perimeter of the polygon and the number of dots, i, inside the polygon.

In the polygon shown, p = 7 and i = 4.



1 The diagram shows the first three triangles of a sequence with i = 0.



The first triangle in the sequence has p = 4. Its area $A = \frac{1}{2} \times base \times height = \frac{1}{2} \times 2 \times 1 = 1$ square.

(a) Complete the table for the first six triangles in this sequence.

	Triangle						
	first	second	third	fourth	fifth	sixth	
A	1						
р	4	6					

[3]

(b) Find a formula for *p* in terms of *A*.

.....[3]

(c) Make *A* the subject of the formula.

......[2]

2 The diagram shows the first three triangles of a sequence with p = 4. The number of dots, *i*, inside the triangle, increases by one each time.

0	0	0	0	0	•	0	0	0	0	Å	0	0	•
•	•	•	٥	•	•	•	Å	•	•		0	•	0
•	•	0	0	Å	0	•	/•\	•	• /	/ ₀ \	•	•	0
•	\wedge	0	•/	/₀∖	\°	•/	•	\ •	•/	0	\ •	0	0
		_			7	L		7	Ļ		7	•	0

(a) The area of the first triangle is 1 square.

Find the area, A, of each of the other three triangles.

(b) Explain how the connection between the increase in *i* and the increase in *A* changes your answer in **Question 1(c)** to give $A = \frac{1}{2}p + i - 1$.

......[1]

3 $A = \frac{1}{2}p + i - 1$ is *Pick's formula*, which works for any polygon.

Write down the smallest possible value for *p*.

.....[1]

4 Show that Pick's formula gives the correct value for the area of this polygon.



[4]

5 Use Pick's formula to find the area of this polygon.



......[2]

6 The area of a quadrilateral is 4 squares.

Find how many pairs of values of *p* and *i* are possible.

..... [4]

7 A polygon has an area of 104 squares.

It is given that the number of dots inside the polygon equals the square of the number of dots on the perimeter of the polygon.

Find if it is possible to draw this polygon.

Question 8 is printed on the next page.

6

Section B MODELLING DRILLING A TUNNEL

You are advised to spend no more than 45 minutes on this section.

This task models the time and the cost of drilling a tunnel through different types of rock.

On the diagram, A is south of B and C is east of B.

AB = 500 metres and BC = 300 metres.



Engineers want to drill a tunnel from *A* to *C*. The tunnel will have one or two straight sections and no curved sections.

8 Calculate the length of the shortest possible tunnel from *A* to *C*. Give your answer correct to the nearest metre.

...... m [3]

9 Write down the length of the tunnel if the engineers drill through as little hard rock as possible.

.....[2]

North B 300 m C SCALE x m 500 m P Hard rock Normal rock

8

10 On the diagram, P is a point which is x metres south of B.

The engineers decide to drill from A to P to C.

Through the normal rock, from A to P, the drill moves forward at 2 metres per hour. Through the hard rock, from P to C, the drill moves forward at 1 metre per hour.

(a) Show that the model for the time in hours, T, that it takes to drill the tunnel, is

$$T = \frac{500 - x}{2} + \sqrt{90\ 000 + x^2} \,.$$

[3]

(b) All the measurements are accurate. Write down a practical reason why the time given by the model may be different from the actual time.

.....[1]

(c) On the diagram, sketch the graph of T against x.



(d) (i) Find, correct to the nearest metre, the position of *P* which gives the minimum time to drill the tunnel.

..... m from *B* [1]

(ii) Find this minimum time correct to the nearest 10 hours.

......[1]

- 11 To drill through the normal rock costs 2 thousand dollars per hour. To drill through the hard rock costs 3 thousand dollars per hour.
 - (a) The total cost of drilling the tunnel is *n* thousand dollars. Write down a model for *n* in terms of *x*.

.....[2]

(b) Find the total cost when P is 412.5 metres north of A.

.....[4]

- 12 The model for the time taken to drill the tunnel is $T = \frac{500 x}{2} + \sqrt{90000 + x^2}$. B and C are fixed points.
 - (a) A is more than 500 m south of B.

When T is a minimum, find how changing the position of A affects the value of x.

.....[2]

(b) AB = d metres

Change the model $T = \frac{500 - x}{2} + \sqrt{90000 + x^2}$ to find a model for the minimum time, *t* hours, in terms of *d*.

Write your answer in its simplest form.

......[2]

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12

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