## Cambridge IGCSE ${ }^{\text {TM }}$



You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a graphic display calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly, including sketches, to gain full marks for correct methods.
- In this paper you will be awarded marks for providing full reasons, examples and steps in your working to communicate your mathematics clearly and precisely.


## INFORMATION

- The total mark for this paper is 40.
- The number of marks for each question or part question is shown in brackets [ ].


## INVESTIGATION PICK'S FORMULA

This investigation is about working out the area of a polygon.
In 1899 the Austrian mathematician Georg Pick found a method to work out the area, $A$, of any polygon that has its vertices (corners) on a square grid.

His method used the number of dots, $p$, on the perimeter of the polygon and the number of dots, $i$, inside the polygon.

In the polygon shown, $p=7$ and $i=4$.


1 The diagram shows the first three rectangles of a sequence with $i=0$.


The second rectangle has $p=6$. Its area, $A$, is 2 squares.
(a) Complete the table for the first six rectangles in the sequence.

|  | Rectangle |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | first | second | third | fourth | fifth | sixth |  |
| $p$ |  | 6 |  |  |  | 14 |  |
| $\frac{1}{2} p$ |  | 3 |  |  |  | 7 |  |
| $A$ |  | 2 |  |  |  | 6 |  |

(b) Write down a formula for $A$ in terms of $p$.

2 The diagram shows the first three triangles of a sequence with $p=4$.


The third triangle has $i=2$.
Its area $A=\frac{1}{2} \times$ base $\times$ height $=\frac{1}{2} \times 2 \times 3=3$ squares.
(a) Find the area of the first two triangles in the sequence.

First triangle $\qquad$

Second triangle $\qquad$
(b) Complete the table for the first six triangles in the sequence.

|  | Triangle |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | first | second | third | fourth | fifth | sixth |  |
| $i$ |  |  | 2 |  |  |  |  |
| $A$ |  |  | 3 |  |  |  |  |
|  |  |  |  |  |  |  |  |

(c) Write down a formula for $A$ in terms of $i$.

3 Your answers to Question 1(b) and Question 2(c) show that the area, $A$, of a polygon relates to $\frac{1}{2} p$ and also to $i$.

The diagram shows polygons $Q, R, S, T$ and $U$.

(a) Complete the table.

|  | Polygon |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $Q$ | $R$ | $S$ | $T$ | $U$ |  |
| $\frac{1}{2} p$ |  | 8 | 9 |  | 4.5 |  |
| $i$ |  | 3 | 6 | 2 |  |  |
| $\frac{1}{2} p+i$ | 7 |  |  | 11 | 8.5 |  |
| $A$ | 6 |  |  |  | 7.5 |  |

(b) Write a formula for $A$ in terms of $p$ and $i$.

4 The answer to Question 3(b) is Pick's formula.
Show that Pick's formula gives the correct value for the area of this polygon.


5 Use Pick's formula to find the area of this polygon.


6 The area of triangle $G$ is 3 squares.
There are 6 dots on the perimeter of the triangle.
(a) Use Pick's formula to find the number of dots inside triangle $G$.
(b) Use your answer to part (a) to draw triangle $G$.


7 The area of quadrilateral $H$ is 4 squares. There are 2 dots inside the quadrilateral.
(a) Use Pick's formula to find the number of dots on the perimeter of quadrilateral $H$.
(b) Use your answer to part (a) to draw quadrilateral $H$.


Question 8 is printed on the next page.

8 (a) For any polygon, give the reason why the value of $p$ is greater than 2.
(b) What is true about the value of $p$ when $A$ is a positive integer?
$\qquad$
(c) The area, $A$, of a polygon is 2 squares.

Use Pick's formula to find all the possible pairs of values for $p$ and $i$.

