Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

MATHEMATICS
0580/41
Paper 4 (Extended)
May/June 2018
2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator
Geometrical instruments
Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to
three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets \([\ ]\) at the end of each question or part question.
The total of the marks for this paper is 130.
1 Adele, Barbara and Collette share $680 in the ratio 9 : 7 : 4.

(a) Show that Adele receives $306.

(b) Calculate the amount that Barbara and Collette each receives.

Barbara $ ............................................

Collette $ ........................................... [3]

(c) Adele changes her $306 into euros (€) when the exchange rate is $1 = €1.125.

Calculate the number of euros she receives.

€ ............................................... [2]

(d) Barbara spends a total of $17.56 on 5 kg of apples and 3 kg of bananas.

Apples cost $2.69 per kilogram.

Calculate the cost per kilogram of bananas.

$ ............................................... [3]

(e) Collette spends half of her share on clothes and \( \frac{1}{5} \) of her share on books.

Calculate the amount she has left.

$ ............................................... [3]
The scale drawing shows two boundaries, $AB$ and $BC$, of a field $ABCD$. The scale of the drawing is 1 cm represents 8 m.

(a) The boundaries $CD$ and $AD$ of the field are each 72 m long.
   (i) Work out the length of $CD$ and $AD$ on the scale drawing.
       ................................... cm [1]
   (ii) Using a ruler and compasses only, complete accurately the scale drawing of the field. [2]

(b) A tree in the field is
    • equidistant from $A$ and $B$
    • equidistant from $AB$ and $BC$.

On the scale drawing, construct two lines to find the position of the tree.
Use a straight edge and compasses only and leave in your construction arcs. [4]
3  (a) The price of a house decreased from $82 500 to $77 500.

Calculate the percentage decrease.

............................................ % [3]

(b) Roland invests $12 000 in an account that pays compound interest at a rate of 2.2% per year.

Calculate the value of his investment at the end of 6 years.
Give your answer correct to the nearest dollar.

$ ............................................. [3]
(a) Describe fully the single transformation that maps

(i) triangle $A$ onto triangle $B$,

....................................................................................................................................................
..................................................................................................................................................... [2]

(ii) triangle $A$ onto triangle $C$,

....................................................................................................................................................
..................................................................................................................................................... [3]

(iii) triangle $A$ onto triangle $D$.

....................................................................................................................................................
..................................................................................................................................................... [3]

(b) On the grid, draw the image of triangle $A$ after an enlargement by scale factor 2, centre $(7, 3)$. [2]
5 (a) Factorise.

(i) \(2mn + m^2 - 6n - 3m\)

................................................ [2]

(ii) \(4y^2 - 81\)

................................................ [1]

(iii) \(t^2 - 6t + 8\)

................................................ [2]

(b) Rearrange the formula to make \(x\) the subject.

\[k = \frac{2m-x}{x}\]

\[x = \ldots\] [4]
(c) Solve the simultaneous equations.
You must show all your working.

\[
\begin{align*}
\frac{1}{2}x - 3y &= 9 \\
5x + y &= 28
\end{align*}
\]

\[
\begin{align*}
x &= \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [3] \\
y &= \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [3]
\end{align*}
\]

(d) \[
\frac{3}{m + 4} - \frac{4}{m} = 6
\]

(i) Show that this equation can be written as \( 6m^2 + 25m + 16 = 0 \).

\[
\begin{align*}
[m] [3]
\end{align*}
\]

(ii) Solve the equation \( 6m^2 + 25m + 16 = 0 \).
Show all your working and give your answers correct to 2 decimal places.

\[
\begin{align*}
m &= \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [4] \\
m &= \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [4]
\end{align*}
\]
6 A solid hemisphere has volume 230 cm$^3$.

(a) Calculate the radius of the hemisphere.

[The volume, $V$, of a sphere with radius $r$ is $V = \frac{4}{3}\pi r^3$.]

.......................................... cm $^3$

(b) A solid cylinder with radius 1.6 cm is attached to the hemisphere to make a toy.

The total volume of the toy is 300 cm$^3$.

(i) Calculate the height of the cylinder.

.......................................... cm $^3$
(ii) A mathematically similar toy has volume 19200 cm$^3$.

Calculate the radius of the cylinder for this toy.

.......................................... cm $^3$
The graph of \( y = 10 - 8x^2 \) for \( -1.5 \leq x \leq 1.5 \) is drawn on the grid.
(a) Write down the equation of the line of symmetry of the graph.

................................................... [1]

(b) On the grid opposite, draw the tangent to the curve at the point where $x = 0.5$. Find the gradient of this tangent.

................................................... [3]

(c) The table shows some values for $y = x^3 + 3x + 4$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>−1.5</th>
<th>−1</th>
<th>−0.5</th>
<th>0</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>−3.9</td>
<td></td>
<td></td>
<td>5.6</td>
<td>8</td>
<td>11.9</td>
<td></td>
</tr>
</tbody>
</table>

(i) Complete the table. [3]

(ii) On the grid opposite, draw the graph of $y = x^3 + 3x + 4$ for $−1.5 \leq x \leq 1.5$. [4]

(d) Show that the values of $x$ where the two curves intersect are the solutions to the equation $x^3 + 8x^2 + 3x - 6 = 0$. [1]

(e) By drawing a suitable straight line, solve the equation $x^3 + 5x + 2 = 0$ for $−1.5 \leq x \leq 1.5$.

$x = .............................................$ [3]
8 (a) The exterior angle of a regular polygon is $x^\circ$ and the interior angle is $8x^\circ$.

Calculate the number of sides of the polygon.

(b) \[ \angle CDB = \ldots \] [3]
P, Q and R are points on the circumference of the circle, centre O. PO is parallel to QR and angle POQ = 48°.

(i) Find angle OPR.

Angle OPR = ........................................... [2]

(ii) The radius of the circle is 5.4 cm.

Calculate the length of the major arc PQ.

................................................. cm [3]
The probability that it will rain tomorrow is $\frac{5}{8}$.

If it rains, the probability that Rafael walks to school is $\frac{1}{6}$.

If it does not rain, the probability that Rafael walks to school is $\frac{7}{10}$.

(a) Complete the tree diagram.

(b) Calculate the probability that it will rain tomorrow and Rafael walks to school.

........................................................................................................... [2]

(c) Calculate the probability that Rafael does not walk to school.

........................................................................................................... [3]
10  (a) In 2017, the membership fee for a sports club was $79.50.
This was an increase of 6% on the fee in 2016.

Calculate the fee in 2016.

$ ...........................................  [3]

(b) On one day, the number of members using the exercise machines was 40, correct to the nearest 10.
Each member used a machine for 30 minutes, correct to the nearest 5 minutes.

Calculate the lower bound for the number of minutes the exercise machines were used on this day.

......................................... min [2]

(c) On another day, the number of members using the exercise machines \( (E) \), the swimming pool \( (S) \) and the tennis courts \( (T) \) is shown on the Venn diagram.

(i) Find the number of members using only the tennis courts.

............................................. [1]

(ii) Find the number of members using the swimming pool.

............................................. [1]

(iii) A member using the swimming pool is chosen at random.

Find the probability that this member also uses the tennis courts and the exercise machines.

............................................. [2]

(iv) Find \( n(T \cap (E \cup S)) \).

............................................. [1]
11 (a) \[ \overrightarrow{OA} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \quad \overrightarrow{AB} = \begin{pmatrix} 8 \\ -7 \end{pmatrix}, \quad \overrightarrow{AC} = \begin{pmatrix} -3 \\ 6 \end{pmatrix} \]

Find

(i) \( |\overrightarrow{OB}|, \)

\[ |\overrightarrow{OB}| = \text{.................................} \quad [3] \]

(ii) \( \overrightarrow{BC}. \)

\[ \overrightarrow{BC} = \begin{pmatrix} \phantom{f} \\ \phantom{p} \end{pmatrix} \quad [2] \]

(b) \[ \text{PQRS is a parallelogram with diagonals PR and SQ intersecting at } X. \]

\[ \overrightarrow{PQ} = a \text{ and } \overrightarrow{PS} = b. \]

Find \( \overrightarrow{QX} \) in terms of \( a \) and \( b. \)

Give your answer in its simplest form.

\[ \overrightarrow{QX} = \text{.................................} \quad [2] \]
(c) \( \mathbf{M} = \begin{pmatrix} 2 & 5 \\ 1 & 8 \end{pmatrix} \)

Calculate

(i) \( \mathbf{M}^2 \),

\[ \mathbf{M}^2 = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \]

(ii) \( \mathbf{M}^{-1} \).

\[ \mathbf{M}^{-1} = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \]
12 Marco is making patterns with grey and white circular mats.

The patterns form a sequence.

Marco makes a table to show some information about the patterns.

<table>
<thead>
<tr>
<th>Pattern number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of grey mats</td>
<td>6</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Total number of mats</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

(a) Complete the table for Pattern 5. [2]

(b) Find an expression, in terms of $n$, for the number of grey mats in Pattern $n$. ................................................ [2]

(c) Marco makes a pattern with 24 grey mats.

Find the total number of mats in this pattern.

................................................ [2]
(d) Marco needs a total of 6 mats to make the first pattern.
He needs a total of 16 mats to make the first two patterns.
He needs a total of \( \frac{1}{6} n^3 + an^2 + bn \) mats to make the first \( n \) patterns.

Find the value of \( a \) and the value of \( b \).