## Cambridge Pre-U

## MATHEMATICS

Paper 3 Applications of Mathematics
May/June 2022
2 hours
You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs
- Write your name, centre number and candidate number on all the work you hand in.
- Do not use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity is needed, use $10 \mathrm{~m} \mathrm{~s}^{-2}$.
- At the end of the examination, fasten all your work together. Do not use staples, paper clips or glue.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Section A: Probability (40 marks)

## You are advised to spend no more than 1 hour on this section.

1 The marks, $x$, awarded by a teacher to 28 students for a particular piece of work are summarised as follows.

$$
\Sigma x=1638, \quad \Sigma x^{2}=108364
$$

(a) Calculate the mean and standard deviation of these marks.

The teacher finds that, compared to other teachers, she has marked too harshly. To bring her marking into line with the other teachers, she adds 4 to the mark of each student.
(b) Write down the mean and standard deviation of the new marks.

2 Two events, $A$ and $B$, are such that $\mathrm{P}(A)=\frac{1}{2}, \mathrm{P}(B)=\frac{7}{10}$ and $\mathrm{P}\left(A \mid B^{\prime}\right)=\frac{2}{3}$.
(a) Find $\mathrm{P}(A \cap B)$.
(b) Find $\mathrm{P}(A \cup B)$.
(c) State, with a reason, whether $A$ and $B$ are mutually exclusive.

3 A discrete random variable $X$ has the following distribution.

| $x$ | 0 | 2 | 4 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{P}(X=x)$ | 0.25 | $p$ | $q$ | 0.4 |

It is given that $\mathrm{E}(X)=3.5$.
(a) Find $p$ and $q$.
(b) Find the variance of $X$.

4 Patients' waiting times at a hospital A\&E department are modelled by a normal distribution with standard deviation 1.8 hours. The waiting time is longer than 7 hours for $10 \%$ of the patients.
(a) Find the mean waiting time.
(b) Find the probability that a randomly chosen patient waits less than 3 hours.
(a) The 11 letters of the word RASPBERRIES are to be arranged.
(i) Find the number of different arrangements.
(ii) Find the number of different arrangements if the 4 vowels are all together.
(b) 4 of the 11 letters of the word RASPBERRIES are chosen at random. Find the probability that the 4 letters can be arranged to spell the word RIPE.

6 A shop sells ball-point pens, of which $3 \%$ are faulty.
(a) (i) A random sample of 20 pens is taken. Find the probability that at most 2 pens are faulty.
(ii) Five random samples of size 20 are taken. Find the probability that exactly three of the samples contain at most 2 pens that are faulty.
(b) A random sample of $n$ pens is taken. Find the smallest value of $n$ such that there is a probability of at least 0.9 that the sample contains at least 1 faulty pen.

## Section B: Mechanics (40 marks)

## You are advised to spend no more than 1 hour on this section.

7 A block of mass 8 kg lies on a rough plane inclined at $30^{\circ}$ to the horizontal. The coefficient of friction between the block and the plane is $\mu$. A force of magnitude 25 N is applied to the block in a direction up the plane and along a line of greatest slope of the plane. This 25 N force is just sufficient to prevent the block sliding down the plane.

Find the value of $\mu$.

8 A light inextensible string connects two particles $A$ and $B$, of masses 0.3 kg and 0.5 kg respectively. $A$ is held with $B$ vertically below $A$. Particle $A$ is released and the system falls. The air resistance acting on $A$ is 0.8 N and the air resistance acting on $B$ is 0.6 N .

Find the acceleration of the particles and the tension in the string.


The diagram shows the velocity-time graph for the motion of a small object, which falls vertically from rest at a point $P$ above the surface of a reservoir. The velocity-time graph consists of two straight line segments.

The downward velocity of the object $t \mathrm{~s}$ after leaving $P$ is $v \mathrm{~m} \mathrm{~s}^{-1}$. The object hits the surface of the reservoir with velocity $V \mathrm{~m} \mathrm{~s}^{-1}$ when $t=0.8$. It reaches the bottom of the reservoir with velocity $5 \mathrm{~m} \mathrm{~s}^{-1}$ when $t=1.4$.
(a) Given that there is no air resistance, find $V$.
(b) Find the depth of the reservoir.
(c) The resistance to the motion of the object in the reservoir has magnitude 6 N . Find the mass of the object.

10 A particle is projected from a point $P$ on horizontal ground with speed $45 \mathrm{~m} \mathrm{~s}^{-1}$ at $\theta^{\circ}$ above the horizontal. The particle reaches the ground again, at point $Q, 4 \mathrm{~s}$ after projection.
(a) Find $\theta$ and the distance $P Q$.
(b) Find the greatest height above the ground reached by the particle.
(c) Find the length of time for which the direction of motion is between $10^{\circ}$ above the horizontal and $10^{\circ}$ below the horizontal.

11 Two particles $A$ and $B$, of masses $m$ and $3 m$ respectively, move on a smooth horizontal surface. $A$ is moving with speed $u$ when it collides directly with $B$, which is at rest. The coefficient of restitution between $A$ and $B$ is $e$. The direction of $A$ is reversed by the collision.
(a) Find, in terms of $u$ and $e$, the speed of $A$ and the speed of $B$ immediately after the collision. [7]

Subsequently, $B$ collides directly with a smooth vertical wall and rebounds. The coefficient of restitution between $B$ and the wall is $\frac{1}{2}$. There is a second collision between $A$ and $B$.
(b) Show that $\frac{1}{3}<e<\frac{3}{5}$.

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