## Cambridge Pre-U

## MATHEMATICS

Paper 2 Pure Mathematics 2
May/June 2022
2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs
- Write your name, centre number and candidate number on all the work you hand in.
- Do not use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do not use staples, paper clips or glue.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].

This document has 4 pages. Any blank pages are indicated.

1 Solve $|2 x-3|=|x-1|$.

2 The points $P$ and $Q$ have coordinates $(1,7)$ and $(9,3)$ respectively.
(a) Find the equation of the perpendicular bisector of $P Q$, giving your answer in the form $y=m x+c$.
(b) Find the area of the triangle enclosed by the $x$-axis, the $y$-axis and the perpendicular bisector of $P Q$.

3 (a) Express $(1-x)(x-4)$ in the form $a-(x-b)^{2}$, where $a$ and $b$ are constants to be found.
(b) State the coordinates of the maximum point of the curve $y=(1-x)(x-4)+\frac{3}{4}$.

4 The points $A$ and $B$ have position vectors $3 \mathbf{i}-\mathbf{j}+2 \mathbf{k}$ and $-2 \mathbf{i}+\mathbf{j}+3 \mathbf{k}$ respectively, relative to the origin $O$. The line $L$ passes through $A$ and $B$.
(a) Given that $L$ is parallel to the vector $10 \mathbf{i}+a \mathbf{j}+b \mathbf{k}$, find the values of $a$ and $b$.
(b) Given also that $L$ is perpendicular to the vector $2 \mathbf{i}+\mathbf{j}+c \mathbf{k}$, find the value of $c$.

5


The diagram shows part of the curve $x y=6$ and part of the line $y=7-x$. Find the exact area of the region enclosed by the curve and the line, shaded on the diagram.

6 Solve the equation $7 \cos \theta-4 \sin \theta=3$ for $0^{\circ}<\theta<360^{\circ}$.

7 A geometric progression, $u_{1}, u_{2}, u_{3}, \ldots$, has first term $u_{1}=a$ and common ratio $r$, where $|r|<1$. The sum to infinity of the series $u_{1}+u_{2}+u_{3}+\ldots$ is 8 .

The sum to infinity of the series consisting of the even-numbered terms, $u_{2}+u_{4}+u_{6}+\ldots$, is 2 .
Determine the value of $a$ and the value of $r$.

8 (a) Find the two complex numbers $z$ which satisfy both $\frac{z}{z^{*}}=\frac{3}{5}+\frac{4}{5} \mathrm{i}$ and $z z^{*}=5$. Give your answers in the form $z=x+\mathrm{i} y$.
(b) Sketch on an Argand diagram the locus of points given by $|w-4|=|w+2 \mathbf{i}|$, for complex numbers $w$.

9 A triangle has sides $p-q, p$ and $p+q$, where $p>q>0$. The largest angle of the triangle is $\alpha$.
(a) Use the cosine rule to show that $\cos \alpha=\frac{p-4 q}{2(p-q)}$.
(b) Given that $p=7$ and $q=1$, find the exact area of the triangle.
(c) Given instead that $\alpha=150^{\circ}$, find an expression for $p$ in terms of $q$.

10 A curve has equation $y=\frac{1-x^{2}}{1+x^{2}}$.
(a) (i) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=-\frac{4 x}{\left(1+x^{2}\right)^{2}}$.
(ii) Find $\frac{\mathrm{d}^{2} y}{\mathrm{~d} x^{2}}$ in terms of $x$, simplifying your answer.
(b) The curve crosses the $x$-axis at two points $A$ and $B$. Show that the tangents to the curve at $A$ and $B$ intersect at a point $C$ on the $y$-axis.
(c) Show that $C$ is a maximum point of the curve.

11 It is given that $k>\ln 5$ and that $\int_{\ln 5}^{k} \frac{10}{\mathrm{e}^{x}-3-4 \mathrm{e}^{-x}} \mathrm{~d} x=\ln 25$. Using the substitution $u=\mathrm{e}^{x}$, or otherwise, determine the exact value of the constant $k$.

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