

# **Cambridge Pre-U**

## MATHEMATICS

Paper 1 Pure Mathematics 1

9794/01

May/June 2022

2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper Graph paper List of formulae (MF20)

### INSTRUCTIONS

- Answer all questions.
- If you have been given an answer booklet, follow the instructions on the front cover of the answer booklet.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number on all the work you hand in.
- Do **not** use an erasable pen or correction fluid.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- At the end of the examination, fasten all your work together. Do **not** use staples, paper clips or glue.

### INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **4** pages. Any blank pages are indicated.

- A sequence  $u_1, u_2, u_3, \dots$  is defined by  $u_1 = 1$  and  $u_{n+1} = \frac{4}{2 u_n}$  for  $n \ge 1$ . 1
  - (a) Write down the values of  $u_2$ ,  $u_3$  and  $u_4$ . [2]

(b) Find 
$$\sum_{n=1}^{50} u_n$$
. [3]

- Solve the equation  $x^{\frac{1}{2}} 6x^{\frac{1}{4}} + 8 = 0$ . 2 [5]
- A complex number z has modulus 4 and argument  $-\frac{1}{3}\pi$ . Giving your answers in the form x + iy, find 3
  - (a) z, [2]

**(b)** 
$$\frac{8}{iz}$$
. [4]

4

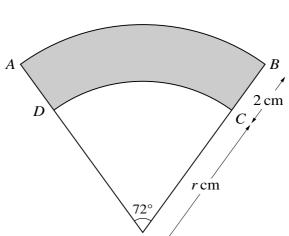
2 cm D C rcm O

The diagram shows a sector AOB of a circle with centre O and radius (r + 2) cm. Angle AOB is 72°. The points C and D lie on OB and OA respectively, and CD is an arc of a circle with centre O and radius r cm. The area of the shaded region ABCD is  $8\pi$  cm<sup>2</sup>.

(a) Express  $72^{\circ}$  in radians, giving your answer in an exact form. [1]

[4]

- (b) Determine the value of *r*.
- The line *L* has equation  $\mathbf{r} = \begin{pmatrix} 3 \\ 10 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$ . The point *A* has coordinates (1, -2, 1) and the point *B* 5 has coordinates (8, 12, -6). The line L intersects the line through A and B at the point C.
  - (a) Find the coordinates of C. [5]
  - (b) Determine the ratio AC : CB, giving the answer in its simplest form. [2]



6 The function f is defined for all real values of x by  $f(x) = (x - 14)^3 + 3$ .

(a) Find 
$$ff(16)$$
. [2]

[3]

[7]

(**b**) Find  $f^{-1}(x)$ , stating its domain.

The curve  $y = (x - 14)^3 + 3$  intersects the line y = x at the point *P*.

- (c) Use the Newton-Raphson method on  $(x 14)^3 + 3 x = 0$ , with  $x_0 = 20$ , to find the coordinates of the point *P*. Show the result of each iteration and give each coordinate correct to 4 significant figures. [3]
- (d) State how the point *P* is related to the curves y = f(x) and  $y = f^{-1}(x)$ . [1]
- 7 (a) Sketch, on a single diagram, the graphs of  $y = \operatorname{cosec} x$  and  $y = \tan x$  for  $0^\circ \le x \le 360^\circ$ . [4]
  - (b) Solve the equation  $\csc x = \tan x$  for  $0^{\circ} \le x \le 360^{\circ}$ . [5]
- 8 (a) (i) Show that  $(\sqrt{a} \sqrt{a+b})(\sqrt{a} + \sqrt{a+b}) = -b.$  [1]

(ii) Hence show that 
$$\frac{1}{\sqrt{a+b}} - \frac{1}{\sqrt{a}} = \frac{-b}{a\sqrt{a+b} + \sqrt{a}(a+b)}$$
. [3]

- (b) Given that  $f(x) = x^{-\frac{1}{2}}$ , use differentiation from first principles to prove that  $f'(x) = -\frac{1}{2}x^{-\frac{3}{2}}$ . [3]
- 9 Find the solution of the differential equation  $\frac{dy}{dx} = y \ln(x+2)$ , given that y = 27 when x = 1. Give your answer in the form f(y) = g(x). [7]

10 The expansion of  $(1 + ax)^n$  contains the terms  $-\frac{2}{3}x$  and  $-\frac{112}{81}x^3$ .

Find the possible values of *a* and *n*.

11 (a) Show that  $\frac{d}{dx}(\sec x \tan x + \ln(\sec x + \tan x)) = 2 \sec^3 x.$  [7] [You may quote the derivatives of sec x and tan x from the list of formulae (MF20).]

(b) The region bounded by the curve  $y = \frac{1}{1-x^2}$ , the *x*-axis and the lines x = 0 and  $x = \frac{1}{2}$  is rotated completely around the *x*-axis. Use the substitution  $x = \sin u$  to show that the volume generated is given by  $\frac{1}{12}\pi(4+3\ln 3)$ . [6]

### **BLANK PAGE**

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.