Cambridge Assessment International Education

Cambridge IGCSE™

ADDITIONAL MATHEMATICS (US) **0459/01**

Paper 1 **For examination from 2020**

MARK SCHEME

Maximum Mark: 80

Specimen

This document has **8** pages. Blank pages are indicated.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles

1. Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2. Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3. Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5. Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6. Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem.
A Accuracy mark, given for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the **M** marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several **B** marks allocated. The notation 'dep' is used to indicate that a particular **M** or **B** mark is dependent on an earlier mark in the scheme.

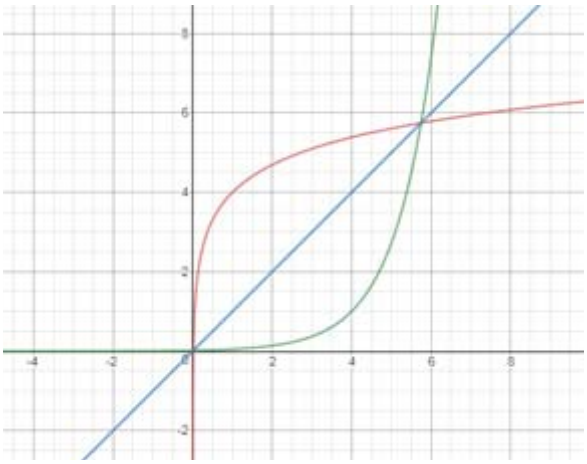
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	special case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	$(x + 2)^2 + (y – 8)^2 = 7^2$ or $x^2 + y^2 + 4x – 16y + 19 = 0$	**B2**	**B1** for $(x + 2)^2 + (y – 8)^2$ or $x^2 + 4x + y^2 + 16y$ oe **B1** for $= 7^2$ or $+ 2^2 + 8^2 – 7^2 = 0$ oe
2(a)	Any two valid reasons e.g. Size of population may make selection of every item impossible Gathering information may necessitate destruction of items e.g. life of a battery	**B2**	**B1 + B1**
2(b)	Every member of population has the same chance of being selected at every stage if random and choice of 1st item immediately rules out approximately 90% of the remaining population oe	**B2**	**B2, 1, 0** Explanation must incorporate the essential idea of random sampling
3	$(z_1 =) \frac{3 + \sqrt{7i^2}}{2}$	**M1**	allow $z =$; allow $\frac{3 \pm \sqrt{7i^2}}{2}$
	$(z_1 =) \frac{3 + i\sqrt{7}}{2}$	**A1**	
	$(z_2 =) \frac{3 - i\sqrt{7}}{2}$	**B1**	**FT** the complex conjugate of *their* z_1 $\frac{3 \pm i\sqrt{7}}{2}$ scores 3 marks
4	$(PS)^2 = (x – 6)^2 + (y – 1)^2$	**B1**	
	$(x – 6)^2 + (y – 1)^2 = (x + 1)^2$	**M1**	
	$x^2 – 12x + 36 + y^2 – 2y + 1 = x^2 + 2x + 1$	**M1**	
	$y(y – 2) = 14x – 36$	**A1**	
5	$\left(5 + 2\sqrt{3}\right)^2 = 37 + 20\sqrt{3}$	**B1**	Seen anywhere
	$\frac{\left(37 + 20\sqrt{3}\right)}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$	**M1**	Or **B1** for a correct pair of simultaneous equations $37 = 2p + 3q$ and $20 = p + 2q$ and **M1** for attempting to solve their equations either by elimination or substitution, condone one error
	$14 + 3\sqrt{3}$	**A2**	**A1 + A1** Answer only scores zero

Question	Answer	Marks	Partial Marks
6	Proving triangle *AED* congruent to triangle *CFB* *AD* = *BC* (parallelogram) *ED* = *FB* (given) ∠*ADE* = ∠*CBF* (alternate angles are equal) Δ*AED* ≡ Δ*CFB* (SAS)	**B3**	**B3, 2, 1, 0** Or triangle *DEC* congruent to *BFA* *AB* = *DC* (parallelogram) ∠*ABF* = ∠*CDE* (alternate angles are equal) Δ*ABF* ≡ Δ*CDE* (SAS)
	∠*AED* = ∠*CFB* (corresponding angles of congruent triangles) ∠*AEF* = ∠*CFE* (each equal to 180 – ∠*AED*) Thus alternate angles are equal	**B1**	Must have reasons ∠*AFB* = ∠*ECD* (corresponding angles of congruent triangles) ∠*AFE* = ∠*FEC* (each equal to 180 – ∠*AFB*) Thus alternate angles are equal
	AE = *FC* (corresponding sides of congruent triangles)	**B1**	*AF* = *EC* (corresponding sides of congruent triangles) Other valid proofs should be awarded appropriate credit
7	$\mathbf{A}^{-1} = k\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$	**B1**	
	$k = \frac{1}{5}$	**B1**	
	$\mathbf{A}^2 = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	**2**	**M1** for attempt to multiply with at least two elements correct correct
	$\mathbf{B} = 2 \times \textit{their}\begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} - \textit{their}\begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	**M1**	
	$\begin{pmatrix} 1 & 15 \\ -5 & 6 \end{pmatrix}$	**A1**	
8(a)	0.97 × 0.04	**M1**	
	0.05 × 0.96	**M1**	
	Summing their products	**M1**	
	0.0868	**A1**	
8(b)	$\textit{their}\ \frac{0.0388}{0.0868}$	**M1**	
	Leading to 0.447(00…)	**A1**	

Question	Answer	Marks	Partial Marks
9	Eliminate x or y	**M1**	
	$4x^2 + 4x - 15 = 0$ or $4y^2 - 28y + 33 = 0$	**A1**	
	Factorise 3 term quadratic	**M1**	
	$x = \frac{3}{2}$ and $-\frac{5}{2}$	**A1**	
	$y = \frac{11}{2}$ and $\frac{3}{2}$	**A1**	
	$\sqrt{4^2 + 4^2}$	**M1**	
	$\sqrt{32}$ or $4\sqrt{2}$ or 5.66	**A1**	
10(a)	$m_{AB} = \frac{1}{5}$	**B1**	
	Uses $m_1 m_2 = -1$ ($= m_{BC} = -5$)	**M1**	
	BC: $y - 5 = -5(x - 6)$ or $5x + y = 35$	**M1**	or gradient $BC = \frac{5}{6 - x_c} = -5$
	C (7,0)	**A1**	
	CD: $y - 0 = \frac{1}{5}(x - 7)$ oe	**A1**	**FT** *their* C and m_{AB}
10(b)	D (1,−1.2)	**B1**	**FT** *their* equation of CD
11(a)	$\sin x = 2\cos x$	**M1**	
	$\tan x = 2$	**M1**	
	63.4	**A1**	
	243.4	**A1**	
11(b)	$2(1 - \cos^2 y) + 3\cos y = 0$	**M1**	
	$2\cos^2 y - 3\cos y - 2 = 0$ $(2\cos y + 1)(\cos y - 2) = 0$	**M1**	or correct use of quadratic formula or completing the square extra solutions within range –1 (once each part)
	$\cos y = -\frac{1}{2}$ 120	**A1**	
	240	**A1**	
12	$(1200\mathbf{i} + 240\mathbf{j}) \div 4$	**M1**	
	their $(300\mathbf{i} + 60\mathbf{j}) - (260\mathbf{i} + 156\mathbf{j})$	**M1**	
	$40\mathbf{i} - 96\mathbf{j}$	**A1**	
	$\sqrt{40^2 + 96^2}$	**M1**	
	104	**A1**	
	$\tan^{-1}\left(\frac{96}{40}\right)$ or $\tan^{-1}\left(\frac{96}{40}\right)$	**M1**	
	157(.4)	**A1**	clear indication of direction

Question	Answer	Marks	Partial Marks
13(a)	4π, 16π, 36π	**B1**	
	4π, $16\pi - 4\pi$, $36\pi - 16\pi$ 4π, 12π, 20π	**M1**	
13(b)	$\frac{1}{9}$ soi	**B1**	
	$\frac{1}{12}$	**B1**	
13(c)	S: 0, 3, 6, 12 $P(S = s)$: $\frac{1}{4}$, $\frac{5}{12}$, $\frac{3}{12}$, $\frac{1}{12}$	**B2**	**B2, 1, 0**
13(d)	*their* $0 \times \frac{1}{4} + 3 \times \frac{5}{12} + 6 \times \frac{3}{12} + 12 \times \frac{1}{12}$ soi	**M1**	
	3.75	**A1**	**FT** *their E(S)*
	75	**A1**	**FT** *their E(S)* × 20
14(a)(i)	$\text{fg}(x) = 3 - \frac{x}{x+2}$	**B1**	
14(a)(ii)	$3 - \frac{x}{x+2} = 10$ $3(x + 2) - x = 10\,(x + 2)$ or better	**M1**	for dealing with fraction appropriately state this mathematically
	leading to $x = -1.75$	**A1**	
14(b)(i)	$\text{h}(x) > 4$	**B1**	for attempting to obtain inverse function
14(b)(ii)	$\text{h}^{-1}(x) = \text{e}^{x-4}$	**M1**	or **M1** for $4 + \ln x = 9$ and
	$\text{h}^{-1}(9) = \text{e}^5$ (≈ 148)	**A1**	**A1** for $x = \text{e}^5$ (≈ 148)
14(b)(iii)	correct graphs 	**B2**	**B1** + **B1** **B1** for each curve
	idea of symmetry	**B1**	

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