Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

MATHEMATICS

Paper 4 (Extended)
May/June 2017
2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator Geometrical instruments
Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO NOT WRITE IN ANY BARCODES.

Answer all questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For \( \pi \), use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets \([\ ]\) at the end of each question or part question.
The total of the marks for this paper is 130.
An energy company charged these prices in 2013.

<table>
<thead>
<tr>
<th>Electricity price</th>
<th>Gas price</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.15 cents per day plus</td>
<td>24.5 cents per day plus</td>
</tr>
<tr>
<td>13.5 cents for each unit used</td>
<td>5.5 cents for each unit used</td>
</tr>
</tbody>
</table>

(a) (i) In 90 days, the Siddique family used 1885 units of **electricity**.

Calculate the total cost, in dollars, of the electricity they used.

$ ........................................ [2]

(ii) In 90 days, the **gas** used by the Khan family cost $198.16.

Calculate the number of units of gas used.

........................................... units [3]

(b) In 2013, the price for each unit of electricity was 13.5 cents.

Over the next 3 years, this price increased exponentially at a rate of 8% per year.

Calculate the price for each unit of electricity after 3 years.

........................................... cents [2]

(c) Over these 3 years, the price for each unit of gas increased from 5.5 cents to 7.7 cents.

(i) Calculate the percentage increase from 5.5 cents to 7.7 cents.

........................................... % [3]
(ii) Over the 3 years, the 5.5 cents increased exponentially by the same percentage each year to 7.7 cents.

Calculate the percentage increase each year.

............................................ % [3]

(d) In 2015, the energy company divided its profits in the ratio shareholders : bonuses : development = 5 : 2 : 6.

In 2015, its profits were $390 million.

Calculate the amount the company gave to shareholders.

$ .................................. million [2]

(e) The share price of the company in June 2015 was $258.25.

This was an increase of 3.3% on the share price in May 2015.

Calculate the share price in May 2015.

$ ............................................... [3]
The time taken for each of 90 cars to complete one lap of a race track is shown in the table.

<table>
<thead>
<tr>
<th>Time (t seconds)</th>
<th>70 &lt; t ≤ 71</th>
<th>71 &lt; t ≤ 72</th>
<th>72 &lt; t ≤ 73</th>
<th>73 &lt; t ≤ 74</th>
<th>74 &lt; t ≤ 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>17</td>
<td>24</td>
<td>21</td>
<td>18</td>
<td>10</td>
</tr>
</tbody>
</table>

(a) Write down the modal time interval.

............. t .......... [1]

(b) Calculate an estimate of the mean time.

.............................................. s [4]

(c) (i) Complete the cumulative frequency table.

<table>
<thead>
<tr>
<th>Time (t seconds)</th>
<th>t ≤ 71</th>
<th>t ≤ 72</th>
<th>t ≤ 73</th>
<th>t ≤ 74</th>
<th>t ≤ 75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cumulative frequency</td>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(ii) On the grid, draw a cumulative frequency diagram to show this information.

(iii) Find the median time.

.............................................. s [1]

(iv) Find the inter-quartile range.

.............................................. s [2]

(d) One lap of the race track measures 3720 metres, correct to the nearest 10 metres.

A car completed the lap in 75 seconds, correct to the nearest second.

Calculate the upper bound for the average speed of this car.

Give your answer in kilometres per hour.

....................................... km/h [4]
(a) (i) Draw the image of triangle $A$ after reflection in the line $x = 4$. [2]

(ii) Draw the image of triangle $A$ after rotation of $90^\circ$ anticlockwise about $(0, 0)$. [2]

(iii) Draw the image of triangle $A$ after translation by the vector $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$. [2]

(b) Describe fully the single transformation that maps triangle $A$ onto triangle $B$.

.............................................................................................................................................................. [3]
..............................................................................................................................................................
..............................................................................................................................................................

(c) Find the matrix that represents the transformation in part (a)(ii).

$$ \begin{pmatrix} 
\end{pmatrix} $$ [2]
(d) Point $P$ has co-ordinates $(4, 1)$.

$F = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ and $G = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ represent transformations.

(i) Find $G(P)$, the image of $P$ after the transformation represented by $G$.

$\left( \ldots \ldots \ldots \ldots , \ldots \ldots \ldots \right) \ [2]\$

(ii) Find $GF(P)$.

$\left( \ldots \ldots \ldots \ldots , \ldots \ldots \ldots \right) \ [3]\$

(iii) Find the matrix $Q$ such that $GQ(P) = P$.

$\left( \begin{array}{c} \ldots \\ \ldots \end{array} \right) \ [3]$
The diagram shows the graph of $y = f(x)$ for $-2.5 \leq x \leq 2$. 
(a) Find \( f(1) \).


............................. [1]

(b) Solve \( f(x) = 3 \).

\[ x = \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots [1] \]

(c) The equation \( f(x) = k \) has only one solution for \(-2.5 \leq x \leq 2\).

Write down the range of values of \( k \) for which this is possible.


............................. [2]

(d) By drawing a suitable straight line, solve the equation \( f(x) = x - 5 \).

\[ x = \ldots \ldots \ldots \ldots \quad \text{or} \quad x = \ldots \ldots \ldots \ldots \quad \text{or} \quad x = \ldots \ldots \ldots \ldots [3] \]

(e) Draw a tangent to the graph of \( y = f(x) \) at the point where \( x = 1 \).

Use your tangent to estimate the gradient of \( y = f(x) \) when \( x = 1 \).


............................. [3]
5 (a) The diagram shows a cylindrical container used to serve coffee in a hotel.

![Diagram of a cylindrical container with dimensions 50 cm height and 18 cm radius.]

The container has a height of 50 cm and a radius of 18 cm.

(i) Calculate the volume of the cylinder and show that it rounds to 50900 cm³, correct to 3 significant figures.

(ii) 30 litres of coffee are poured into the container.

Work out the height, \( h \), of the empty space in the container.

\[ h = \text{... cm} \]
(iii) Cups in the shape of a hemisphere are filled with coffee from the container. The radius of a cup is 3.5 cm.

\[
\text{NOT TO SCALE}
\]

Work out the maximum number of these cups that can be completely filled from the 30 litres of coffee in the container.

[The volume, \(V\), of a sphere with radius \(r\) is \(V = \frac{4}{3} \pi r^3\).]

................................................. [4]

(b) The hotel also uses glasses in the shape of a cone.

\[
\text{NOT TO SCALE}
\]

The capacity of each glass is 95 cm\(^3\).

(i) Calculate the radius, \(r\), and show that it rounds to 3.3 cm, correct to 1 decimal place.

[The volume, \(V\), of a cone with radius \(r\) and height \(h\) is \(V = \frac{1}{3} \pi r^2 h\).]

......................................... [3]

(ii) Calculate the curved surface area of the cone.

[The curved surface area, \(A\), of a cone with radius \(r\) and slant height \(l\) is \(A = \pi rl\).]

......................................... cm\(^2\) [4]
6 (a) Expand the brackets and simplify.

(i) \(4(2x + 5) - 5(3x - 7)\) ............................................... [2]

(ii) \((x - 7)^2\) ............................................... [2]

(b) Solve.

(i) \(\frac{2x}{3} + 5 = -7\) ................................................ [3]

\[ x = \] ............................................................... [3]

(ii) \(4x + 9 = 3(2x - 7)\) ................................................ [3]

\[ x = \] ............................................................... [3]

(iii) \(3x^2 - 1 = 74\) ................................................ [3]

\[ x = \] or \(x = \) ........................................... [3]
7  A line joins the points $A(-3, 8)$ and $B(2, -2)$.

(a) Find the co-ordinates of the midpoint of $AB$.

\[(..................... , .....................) \quad [2]\]

(b) Find the equation of the line through $A$ and $B$.
Give your answer in the form $y = mx + c$.

\[y = ......................... \quad [3]\]

(c) Another line is parallel to $AB$ and passes through the point $(0, 7)$.
Write down the equation of this line.

\[......................... \quad [2]\]

(d) Find the equation of the line perpendicular to $AB$ which passes through the point $(1, 5)$.
Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

\[......................... \quad [4]\]
8 (a)

$A, B$ and $C$ are three towns.
The bearing of $B$ from $A$ is $110^\circ$.
The bearing of $C$ from $B$ is $280^\circ$.
$AC = 38 \text{ km}$ and $AB = 50 \text{ km}$.

(i) Find the bearing of $A$ from $B$.

.................................................[2]

(ii) Calculate angle $BAC$.

Angle $BAC = .............................................[5]

(iii) A road is built from $A$ to join the straight road $BC$.

Calculate the shortest possible length of this new road.

.......................................... km [3]
(b) Town A has a rectangular park.
The length of the park is $x$ m.
The width of the park is $25$ m shorter than the length.
The area of the park is $2200$ m$^2$.

(i) Show that $x^2 - 25x - 2200 = 0$.

(ii) Solve $x^2 - 25x - 2200 = 0$.
Show all your working and give your answers correct to 2 decimal places.

$x = ..................$ or $x = ..................$ [4]
(a) The $n$th term of a sequence is $8n - 3$.

(i) Write down the first two terms of this sequence.

........................................ , ........................................ [1]

(ii) Show that the number 203 is not in this sequence.

(b) Find the $n$th term of these sequences.

(i) 13, 19, 25, 31, ...

..................................................... [2]

(ii) 4, 8, 14, 22, ...

..................................................... [2]

(c) ...

The second term of this sequence is 20 and the third term is 50.
The rule for finding the next term in this sequence is subtract $y$ then multiply by 5.

Find the value of $y$ and work out the first term of this sequence.

$y =$ ...................................................

First term = ........................................... [4]