## Cambridge Pre-U

## MATHEMATICS

Paper 1 Pure Mathematics 1
For examination from 2020
SPECIMEN PAPER 2 hours

You must answer on the answer booklet/paper.

You will need: Answer booklet/paper
Graph paper
List of formulae (MF20)

## INSTRUCTIONS

- Answer all questions.
- Follow the instructions on the front cover of the answer booklet. If you need additional answer paper, ask the invigilator for a continuation booklet.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].

1 A circle has equation $(x-4)^{2}+(y+7)^{2}=64$.
(a) Write down the coordinates of the centre and the radius of the circle.

Two points, $A$ and $B$, lie on the circle and have coordinates $(4,1)$ and $(12,-7)$ respectively.
(b) Find the coordinates of the midpoint of the chord $A B$.

2 The equation of a curve is $y=x^{3}-2 x^{2}-4 x+3$.
(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$.
(b) Hence find the coordinates of the stationary points on the curve.

3 Let $\mathrm{f}(x)=x^{2}$ and $\mathrm{g}(x)=7 x-2$ for all real values of $x$.
(a) Give a reason why f has no inverse function.
(b) Write down an expression for $\operatorname{gf}(x)$.
(c) Find $\mathrm{g}^{-1}(x)$.
(d) Explain the relationship between the graph of $y=\mathrm{g}(x)$ and $y=\mathrm{g}^{-1}(x)$.

4 (a) Show that $x=2$ is a root of the equation $2 x^{3}-x^{2}-15 x+18=0$.
(b) Hence solve the equation $2 x^{3}-x^{2}-15 x+18=0$.

5 The coefficient of $x^{3}$ in the expansion of $(2+a x)^{5}$ is 10 times the coefficient of $x^{2}$ in $\left(1+\frac{a x}{3}\right)^{4}$. Find $a$.

6 Solve the simultaneous equations

$$
\begin{equation*}
x+y=1, \quad x^{2}-2 x y+y^{2}=9 . \tag{6}
\end{equation*}
$$

7 (a) Express $\frac{8 x-1}{(2 x-1)(x+1)}$ in the form $\frac{A}{2 x-1}+\frac{B}{x+1}$ where $A$ and $B$ are constants.
(b) Hence show that $\int_{2}^{5} \frac{8 x-1}{(2 x-1)(x+1)} \mathrm{d} x=\ln 24$.

8 Given that the equation $x^{3}+2 x-7=0$ has a root between $x=1$ and $x=2$, use the Newton-Raphson formula with $x_{\mathrm{o}}=1$ to find this root correct to 3 decimal places.

9 The complex number $3-4 \mathrm{i}$ is denoted by $z$. Giving your answers in the form $x+\mathrm{i} y$, and showing clearly how you obtain them, find
(a) $2 z+z^{*}$,
(b) $\frac{5}{z}$.
(c) Show $z$ and $z^{*}$ on an Argand diagram.

10 (a) Prove that $\cot \theta+\frac{\sin \theta}{1+\cos \theta}=\operatorname{cosec} \theta$.
(b) Hence solve the equation $\cot \left(\theta+\frac{\pi}{4}\right)+\frac{\sin \left(\theta+\frac{\pi}{4}\right)}{1+\cos \left(\theta+\frac{\pi}{4}\right)}=\frac{5}{2}$ for $0 \leqslant \theta \leqslant 2 \pi$.

11 An arithmetic progression has first term $a$ and common difference $d$. The first, ninth and fourteenth terms are, respectively, the first three terms of a geometric progression with common ratio $r$, where $r \neq 1$.
(a) Find $d$ in terms of $a$ and show that $r=\frac{5}{8}$.
(b) Find the sum to infinity of the geometric progression in terms of $a$.

12 (a) Use integration by parts to show that $\int \ln x \mathrm{~d} x=x \ln x-x+c$.
(b) Find
(i) $\int(\ln x)^{2} \mathrm{~d} x$,
(ii) $\int \frac{\ln (\ln x)}{x} \mathrm{~d} x$.

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