



Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS (US)

0459/02

Paper 2

For Examination from 2019

MARK SCHEME

2 hours

Maximum Mark: 80

Specimen

This document consists of **10** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

<p>GENERIC MARKING PRINCIPLE 1:</p> <p>Marks must be awarded in line with:</p> <ul style="list-style-type: none"> ● the specific content of the mark scheme or the generic level descriptors for the question ● the specific skills defined in the mark scheme or in the generic level descriptors for the question ● the standard of response required by a candidate as exemplified by the standardisation scripts.
<p>GENERIC MARKING PRINCIPLE 2:</p> <p>Marks awarded are always whole marks (not half marks, or other fractions).</p>
<p>GENERIC MARKING PRINCIPLE 3:</p> <p>Marks must be awarded positively:</p> <ul style="list-style-type: none"> ● marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate ● marks are awarded when candidates clearly demonstrate what they know and can do ● marks are not deducted for errors ● marks are not deducted for omissions ● answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
<p>GENERIC MARKING PRINCIPLE 4:</p> <p>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.</p>
<p>GENERIC MARKING PRINCIPLE 5:</p> <p>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).</p>

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark, and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- It implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note. B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2. –1 each error. A mark is deducted from the total mark available up to the maximum mark available for that question. The minimum mark awarded is zero e.g., if a candidate makes 3 errors in a question worth 2 marks they score zero.
- The following abbreviations may be used in a mark scheme.
AG “Answer given” on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
cao “Correct answer only” (emphasizing that no “follow through” from a previous error is allowed).
isw “Ignore subsequent working”.
oe “Or equivalent”.
sc “Special case”. Awarded for some questions where e.g., the candidate has not used the method specified but a different, correct, method leading to the correct answer.
soi “Seen or implied”.

Question	Answer	Marks	Guidance
1	$\frac{x+1}{x^2-3}\sqrt{x^3+x^2+4x+6}$ $\frac{-3x}{x^2+7x+6}$ $\frac{x^2-3}{7x+9}$ <p>$a = 1, b = 7, c = 9$</p>	M1 A2	for getting as far as $x^2 + 7x$ or for a reasonable attempt at a valid method A2, 1, 0 -1 each error
2	$\vec{PQ} = \begin{pmatrix} 9 \\ 20 \end{pmatrix} - \begin{pmatrix} 6 \\ 7 \end{pmatrix}$ $\vec{QR} = \begin{pmatrix} 12 \\ 52 \end{pmatrix}$ $\vec{OR} = \begin{pmatrix} 21 \\ 72 \end{pmatrix}$ $ \vec{OR} = \sqrt{21^2 + 72^2} = 75$ <p>Unit vector = $\frac{1}{75} \begin{pmatrix} 21 \\ 72 \end{pmatrix}$ oe</p>	M1 A1 B1ft M1 A1	
3	$\frac{(3-x)^{-2}}{3} + (3-x)^{\frac{1}{3}}$ $\frac{1}{(3-x)^{\frac{1}{3}}}$ $(3-x)^{-1} + 1$ <p>their $(3-x)^{-1} = \frac{2}{3} - 1$ or better</p> <p>6</p>	B1 B2 M1 A1	soi B1 + B1

Question	Answer	Marks	Guidance
4	$f(3) = \frac{1}{2}f(2) + f(1)$	B1	soi
	$28.5 = \frac{1}{2} \times 17 + f(1)$	M1	
	$b = 20$	A1	
	$17 = \frac{1}{2} \times \textit{their } 20 + f(0)$	M1	
	$a = 7$	A1	

Question	Answer	Marks	Guidance
5(a)(i)	Accuracy of plots	B2	B2, 1, 0 –1 each error, allow tolerance ± 1 mm
5(a)(ii)	$\bar{o} = 26$ $\bar{w} = 55$ Reasonable line of best fit through <i>their</i> (26, 55) Linear equation with reasonable slope	B2 B1ft B1ft	B1 + B1
5(a)(iii)	Approximately 27	B1ft	ft <i>their</i> line provided line reasonable

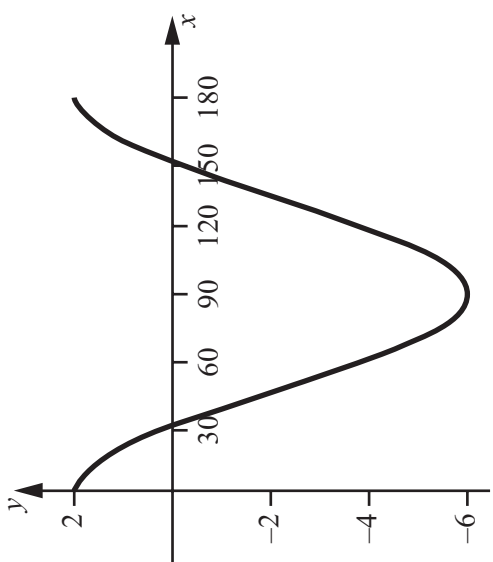
Question	Answer	Marks	Guidance
6(a)	Express as powers of 2 or 4 or 8 Applies rules of indices	M1 DM1	$[2x - (5 - x) = 4x - 3(x - 3)]$
	7	A1	
6(b)	$\lg(2y + 10) + \lg y = \lg \{y(2y + 10)\}$ or $2 = \lg 100$ $2y^2 + 10y = 100$ oe	B1 B1	
	5 only	B1	

Question	Answer	Marks	Guidance
7	Either $(300 \ 240)$, $\begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$, $\begin{pmatrix} 5 \\ 7 \\ 10 \end{pmatrix}$ or $(5 \ 7 \ 10)$, $\begin{pmatrix} 0.6 & 0.5 \\ 0.3 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}$, $\begin{pmatrix} 300 \\ 240 \end{pmatrix}$	B2	B2, 1, 0 The order of writing down is not important for B2, provided the matrices they have written down are conformable if they are written in the correct order. –1 each incorrect matrix.
	First product, either $(300 \ 240)$ $\begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$ or $\begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.5 & 0.4 & 0.1 \end{pmatrix}$ $\begin{pmatrix} 5 \\ 7 \\ 10 \end{pmatrix}$ or $(5 \ 7 \ 10)$ $\begin{pmatrix} 0.6 & 0.5 \\ 0.3 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}$ or $\begin{pmatrix} 0.6 & 0.5 \\ 0.3 & 0.4 \\ 0.1 & 0.1 \end{pmatrix}$ $\begin{pmatrix} 300 \\ 240 \end{pmatrix}$	M1	selection of pair of matrices conformable for multiplication and an attempt to multiply
	$(300 \ 186 \ 54)$ or $\begin{pmatrix} 6.1 \\ 6.3 \end{pmatrix}$ or $(6.1 \ 6.3)$ or $\begin{pmatrix} 300 \\ 186 \\ 54 \end{pmatrix}$	A1	
	Second product, either $(300 \ 186 \ 54)$ $\begin{pmatrix} 5 \\ 7 \\ 10 \end{pmatrix}$ or $(300 \ 240)$ $\begin{pmatrix} 6.1 \\ 6.3 \end{pmatrix}$ or $(6.1 \ 6.3)$ $\begin{pmatrix} 300 \\ 240 \end{pmatrix}$ or $(5 \ 7 \ 10)$ $\begin{pmatrix} 300 \\ 186 \\ 54 \end{pmatrix}$	M1	selection of their first product and remaining matrix, conformable for multiplication and an attempt to multiply
	3342	A1	if M0 then SC2 for correct arithmetic method leading to 3342

Question	Answer	Marks	Guidance
8(i)	$\frac{3}{8} = \frac{5}{8} - \cos A \sin B$	M1	
	$\frac{1}{4}$ oe	A1	
8(ii)	$\frac{5}{8} + \text{their } \frac{1}{4}$	M1	
	$\frac{7}{8}$	A1	
8(iii)	$\frac{\sin A}{\tan B} = \frac{\sin A}{\cos A} \times \frac{\sin B}{\cos B}$	M1	or $\cos A = \text{their } \frac{1}{4 \sin B}$ and $\sin A = \frac{5}{8 \cos B}$ and an attempt at $\frac{\sin A}{\cos A}$ oe
	$= \frac{\sin A \cos B}{\cos A \sin B}$	A1	or $\tan A = \frac{5}{8 \cos B} \times 4 \sin B$ oe
	$= \frac{5}{8} \times \frac{1}{4} = 2.5$	A1	or $\tan A = \frac{5}{2} \tan B$ therefore $\frac{\tan A}{\tan B} = \frac{5}{2}$ oe

Question	Answer	Marks
9(i) & (ii)	For correctly standardizing once in either (i) or (ii)	M1
	$z = \frac{27-24}{4}$ or $z = \frac{20-24}{4}$ or $z = \frac{25-24}{4}$	A1
	0.2266	A1
	0.5987	A1
	0.1587	A1
	0.5987 – 0.1587 oe	M1
0.44	A1	

Question	Answer	Marks	Guidance
10(i)	$-3 - 2i$	B1	
10(ii)	<p>$L \times$</p> <p>$M \times$</p> <p>$N \times$</p>	B2	B2, 1, 0 ft <i>their</i> (i). -1 for each error or omission.
10(iii)	<p>MN is parallel to the Re axis; LM is parallel to the Im axis, therefore $LM \perp MN$ and LMN is right-angled at M</p>	B2	B2, 1, 0
10(iv)	<p>$-15 + 3i + 10i - 2i^2$</p> <p>Correct use of $i^2 = -1$ in <i>their</i> expression</p> <p>$-13 + 13i$</p>	M1 M1 A1	for at least 3 out of 4 correct soi
10(v)	<p>$\left(13\sqrt{2}, \frac{3}{4}\pi\right)$ oe</p>	B2ft	B1ft + B1ft ft <i>their</i> (iv)

Question	Answer	Marks	Guidance
11(i)	Amplitude 4, Period 180	B2	B1 + B1
11(ii)	$x = 90, y = -6$ oe	B2	B1 + B1
11(iii)	$2x = \cos^{-1}\left(\frac{2}{4}\right)$ or better $x = 30$ $x = 150$	M1 A1 A1	
11(iv)		B2	B2, 1, 0 2 and -6 marked on the y-axis 30 and 150 marked on the x-axis correct shape between 0 and 180 ignore any extra sections outside 0 to 180.
11(v)	Section between $(x =) 30$ and 150 reflected in x-axis oe	B1	any portion of the graph below the x-axis will be reflected above.

Question	Answer	Marks	Guidance
12(a)	$x = -1$ or 7 or $-\frac{1}{2}$ seen	M1	for attempt to find a root
	Either $(x + 1)(2x^2 - 13x - 7)$	M1	for attempt to find quadratic factor
	or $(x - 7)(2x^2 + 3x + 1)$	A1	for correct quadratic factor
	or $(2x + 1)(x^2 - 6x - 7)$	M1	for attempt to factorize their quadratic factor
	leading to $(x + 1)(x - 7)(2x + 1)$	A1	for correct factorization Must be 3 term
12(b)	Find $f(2)$ or $f(-3)$	M1	or long division as far as remainder
	$8 + 4a - 30 + b = 0$ or $4a + b = 22$	A1	
	$-27 + 9a + 45 + b = 75$ or $9a + b = 57$	A1	
	Solve simultaneous equations	M1	
	$a = 7, b = -6$	A1	