



Cambridge Assessment International Education
Cambridge International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS (US)

0459/01

Paper 1

For Examination from 2019

MARK SCHEME

2 hours

Maximum Mark: 80

Specimen

This document consists of **9** printed pages and **1** blank page.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

<p>GENERIC MARKING PRINCIPLE 1:</p> <p>Marks must be awarded in line with:</p> <ul style="list-style-type: none"> ● the specific content of the mark scheme or the generic level descriptors for the question ● the specific skills defined in the mark scheme or in the generic level descriptors for the question ● the standard of response required by a candidate as exemplified by the standardisation scripts.
<p>GENERIC MARKING PRINCIPLE 2:</p> <p>Marks awarded are always whole marks (not half marks, or other fractions).</p>
<p>GENERIC MARKING PRINCIPLE 3:</p> <p>Marks must be awarded positively:</p> <ul style="list-style-type: none"> ● marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate ● marks are awarded when candidates clearly demonstrate what they know and can do ● marks are not deducted for errors ● marks are not deducted for omissions ● answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
<p>GENERIC MARKING PRINCIPLE 4:</p> <p>Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.</p>
<p>GENERIC MARKING PRINCIPLE 5:</p> <p>Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).</p>

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

- Marks are of the following three types:
 - M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark, and in some cases an M mark can be implied from a correct answer.
 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Accuracy mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- It implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note. B2 or A2 means that the candidate can earn 2 or 0.
B2, 1, 0 means that the candidate can earn anything from 0 to 2. –1 each error. A mark is deducted from the total mark available up to the maximum mark available for that question. The minimum mark awarded is zero e.g., if a candidate makes 3 errors in a question worth 2 marks they score zero.
- The following abbreviations may be used in a mark scheme.
AG “Answer given” on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid).
cao “Correct answer only” (emphasizing that no “follow through” from a previous error is allowed).
isw “Ignore subsequent working”.
oe “Or equivalent”.
sc “Special case”. Awarded for some questions where e.g., the candidate has not used the method specified but a different, correct, method leading to the correct answer.
soi “Seen or implied”.

Question	Answer	Marks	Guidance
1	$(x + 2)^2 + (y - 8)^2 = 7^2$ or $x^2 + y^2 + 4x - 16y + 19 = 0$	B2	B1 for $(x + 2)^2 + (y - 8)^2$ or $x^2 + 4x + y^2 + 16y$ oe B1 for $= 7^2$ or $+ 2^2 + 8^2 - 7^2 = 0$ oe
Question	Answer	Marks	
2(a)	Any two valid reasons e.g. Size of population may make selection of every item impossible Gathering information may necessitate destruction of items e.g. life of a battery	B2	B1 + B1
2(b)	Every member of population has the same chance of being selected at every stage if random and choice of 1st item immediately rules out approximately 90% of the remaining population oe	B2	B2, 1, 0 Explanation must incorporate the essential idea of random sampling.
Question	Answer	Marks	
3	$(z_1 =) \frac{3 + \sqrt{7i^2}}{2}$ $(z_1 =) \frac{3 + i\sqrt{7}}{2}$ $(z_2 =) \frac{3 - i\sqrt{7}}{2}$	M1 A1 B1ft	allow $z =$; allow $\frac{3 \pm \sqrt{7i^2}}{2}$ ft the complex conjugate of their z_1 $\frac{3 \pm i\sqrt{7}}{2}$ scores 3 marks
Question	Answer	Marks	
4	$(PS)^2 = (x - 6)^2 + (y - 1)^2$ $(x - 6)^2 + (y - 1)^2 = (x + 1)^2$ $x^2 - 12x + 36 + y^2 - 2y + 1 = x^2 + 2x + 1$ $y(y - 2) = 14x - 36$ AG	B1 M1 M1 A1	

Question	Answer	Marks	Guidance
5	$(5 + 2\sqrt{3})^2 = 37 + 20\sqrt{3}$	B1	Seen anywhere
	$\frac{(37 + 20\sqrt{3})}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$	M1	Or B1 for a correct pair of simultaneous equations $37 = 2p + 3q$ and $20 = p + 2q$ and M1 for attempting to solve their equations either by elimination or substitution, condone one error.
	$14 + 3\sqrt{3}$	A2	A1 + A1 Answer only scores zero.
Question	Answer	Marks	Guidance
6	Proving triangle AED congruent to triangle CFB $AD = BC$ (parallelogram) $ED = FB$ (given) $\angle ADE = \angle CBF$ (alternate angles are equal) $\triangle AED \cong \triangle CFB$ (SAS)	B3	B3, 2, 1, 0 Or triangle DEC congruent to BFA $AB = DC$ (parallelogram) $\angle ABF = \angle CDE$ (alternate angles are equal) $\triangle ABF \cong \triangle CDE$ (SAS)
	$\angle AED = \angle CFB$ (corresponding angles of congruent triangles) $\angle AEF = \angle CFE$ (each equal to $180 - \angle AED$) Thus alternate angles are equal	DB1	Must have reasons $\angle AFB = \angle ECD$ (corresponding angles of congruent triangles) $\angle AFE = \angle FEC$ (each equal to $180 - \angle AFB$) Thus alternate angles are equal
	$AE = FC$ (corresponding sides of congruent triangles)	DB1	$AF = EC$ (corresponding sides of congruent triangles) Other valid proofs should be awarded appropriate credit

Question	Answer	Marks	Guidance
7	$A^{-1} = k \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$	B1	
	$k = \frac{1}{5}$	B1	
	$A^2 = \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	2	M1A1 attempt to multiply with at least two elements correct correct
	$B = 2 \times \text{their} \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix} - \text{their} \begin{pmatrix} 1 & -9 \\ 3 & -2 \end{pmatrix}$	M1	
	$\begin{pmatrix} 1 & 15 \\ -5 & 6 \end{pmatrix}$	A1	
Question	Answer	Marks	
8(i)	0.97×0.04	M1	
	0.05×0.96	M1	
	Summing their products	M1	
	0.0868	A1	
8(ii)	$\frac{0.0388}{\text{their } 0.0868}$	M1	
	0.447(00...) A.G.	A1	

Question	Answer	Marks	Guidance
9	Eliminate x or y	M1	
	$4x^2 + 4x - 15 = 0$ or $4y^2 - 28y + 33 = 0$	A1	
	Factorise 3 term quadratic	M1	
	$x = \frac{3}{2}$ and $-\frac{5}{2}$	A1	
	$y = \frac{11}{2}$ and $\frac{3}{2}$	A1	
	$\sqrt{4^2 + 4^2}$	M1	
	$\sqrt{32}$ or $4\sqrt{2}$ or 5.66	A1	

Question	Answer	Marks	Guidance
10(i)	$m_{AB} = \frac{1}{5}$	B1	
	Uses $m_1 m_2 = -1$ ($= m_{BC} = -5$)	M1	
	BC: $y - 5 = -5(x - 6)$ or $5x + y = 35$	M1	or gradient $BC = \frac{5}{6 - x_c} = -5$
	$C(7,0)$	A1	
	CD: $y - 0 = \frac{1}{5}(x - 7)$ oe	A1ft	ft their C and m_{AB}
10(ii)	$D(1, -1.2)$	B1ft	ft their equation of CD

Question	Answer	Marks	Guidance
11(a)	$\sin x = 2\cos x$	M1	
	$\tan x = 2$	M1	
	63.4	A1	
	243.4	A1	

Question	Answer	Marks	Guidance
11(b)	$2(1 - \cos^2 y) + 3\cos y = 0$	M1	
	$2\cos^2 y - 3\cos y - 2 = 0$ $(2\cos y + 1)(\cos y - 2) = 0$	M1	or correct use of quadratic formula or completing the square extra solutions within range -1 (once each part)
	$\cos y = -\frac{1}{2}$ 120	A1	
	240	A1	

Question	Answer	Marks	Guidance
12	$(1200\mathbf{i} + 240\mathbf{j}) \div 4$	M1	
	<i>their</i> $(300\mathbf{i} + 60\mathbf{j}) - (260\mathbf{i} + 156\mathbf{j})$	M1	
	$40\mathbf{i} - 96\mathbf{j}$	A1	
	$\sqrt{40^2 + 96^2}$	M1	
	104	A1	
	$\tan^{-1}\left(\frac{96}{40}\right)$ or $\tan^{-1}\left(\frac{96}{40}\right)$	M1	
	157(.4)	A1	clear indication of direction

Question	Answer	Marks	Guidance
13(i)	$4\pi, 16\pi, 36\pi$	B1	
	$4\pi, 16\pi - 4\pi, 36\pi - 16\pi$ $4\pi, 12\pi, 20\pi$	M1	
13(ii)	$\frac{1}{9} \text{ soi}$	B1	
	$\frac{1}{12}$	B1	

Question	Answer	Marks	Guidance										
13(iii)	<table border="1"> <tr> <td>S</td> <td>0</td> <td>3</td> <td>6</td> <td>12</td> </tr> <tr> <td>$P(S = s)$</td> <td>$\frac{1}{4}$</td> <td>$\frac{5}{12}$</td> <td>$\frac{3}{12}$</td> <td>$\frac{1}{12}$</td> </tr> </table>	S	0	3	6	12	$P(S = s)$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{3}{12}$	$\frac{1}{12}$	B2	B2, 1, 0
S	0	3	6	12									
$P(S = s)$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{3}{12}$	$\frac{1}{12}$									
13(iv)	<p>their $0 \times \frac{1}{4} + 3 \times \frac{5}{12} + 6 \times \frac{3}{12} + 12 \times \frac{1}{12}$ soi</p> <p>3.75</p> <p>75</p>	M1											
		A1	ft <i>their</i> E(S)										
		A1ft											

Question	Answer	Marks	Guidance
14(a)(i)	$fg(x) = 3 - \frac{x}{x+2}$	B1	
14(a)(ii)	$3 - \frac{x}{x+2} = 10$ $3(x+2) - x = 10(x+2)$ or better leading to $x = -1.75$	M1	for dealing with fraction appropriately state this mathematically
14(b)(i)	$h(x) > 4$	A1	
14(b)(ii)	$h^{-1}(x) = e^{x-4}$ $h^{-1}(9) = e^5 (\approx 148)$	B1	for attempting to obtain inverse function
14(b)(iii)	correct graphs	M1	or M1 for $4 + \ln x = 9$ and
	idea of symmetry	A1	A1 for $x = e^5 (\approx 148)$
		B2	B1+B1
		B1	B1 for each curve

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