Notation List

For Cambridge International Mathematics Qualifications

For use from 2020
Mathematical notation

Examinations for CIE syllabuses may use relevant notation from the following list.

1 Set notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>∈</td>
<td>is an element of</td>
</tr>
<tr>
<td>∉</td>
<td>is not an element of</td>
</tr>
<tr>
<td>{x_1, x_2, \ldots}</td>
<td>the set with elements x_1, x_2, \ldots</td>
</tr>
<tr>
<td>{x : \ldots}</td>
<td>the set of all x such that \ldots</td>
</tr>
<tr>
<td>n(A)</td>
<td>the number of elements in set A</td>
</tr>
<tr>
<td>∅</td>
<td>the empty set</td>
</tr>
<tr>
<td>∈</td>
<td>the universal set</td>
</tr>
<tr>
<td>\cup</td>
<td>the universal set (for 0607 IGCSE International Mathematics)</td>
</tr>
<tr>
<td>A'</td>
<td>the complement of the set A</td>
</tr>
<tr>
<td>\mathbb{N}</td>
<td>the set of natural numbers, {1, 2, 3, \ldots}</td>
</tr>
<tr>
<td>\mathbb{Z}</td>
<td>the set of integers, {0, \pm1, \pm2, \pm3, \ldots}</td>
</tr>
<tr>
<td>\mathbb{Q}</td>
<td>the set of rational numbers, \left{ \frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0 \right}</td>
</tr>
<tr>
<td>\mathbb{R}</td>
<td>the set of real numbers</td>
</tr>
<tr>
<td>\mathbb{C}</td>
<td>the set of complex numbers</td>
</tr>
<tr>
<td>(x, y)</td>
<td>the ordered pair x, y</td>
</tr>
<tr>
<td>≤</td>
<td>is a subset of</td>
</tr>
<tr>
<td>⊂</td>
<td>is a proper subset of</td>
</tr>
<tr>
<td>\cup</td>
<td>union</td>
</tr>
<tr>
<td>\cap</td>
<td>intersection</td>
</tr>
<tr>
<td>[a, b]</td>
<td>the closed interval {x \in \mathbb{R} : a \leq x \leq b}</td>
</tr>
<tr>
<td>[a, b)</td>
<td>the interval {x \in \mathbb{R} : a \leq x &lt; b}</td>
</tr>
<tr>
<td>(a, b]</td>
<td>the interval {x \in \mathbb{R} : a &lt; x \leq b}</td>
</tr>
<tr>
<td>(a, b)</td>
<td>the open interval {x \in \mathbb{R} : a &lt; x &lt; b}</td>
</tr>
<tr>
<td>(S, \circ)</td>
<td>the group consisting of the set S with binary operation \circ</td>
</tr>
</tbody>
</table>

2 Miscellaneous symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>=</td>
<td>is equal to</td>
</tr>
<tr>
<td>≠</td>
<td>is not equal to</td>
</tr>
<tr>
<td>≡</td>
<td>is identical to or is congruent to</td>
</tr>
<tr>
<td>≈</td>
<td>is approximately equal to</td>
</tr>
<tr>
<td>~</td>
<td>is distributed as</td>
</tr>
<tr>
<td>≅</td>
<td>is isomorphic to</td>
</tr>
<tr>
<td>∝</td>
<td>is proportional to</td>
</tr>
<tr>
<td>&lt;</td>
<td>is less than</td>
</tr>
<tr>
<td>≤</td>
<td>is less than or equal to</td>
</tr>
<tr>
<td>&gt;</td>
<td>is greater than</td>
</tr>
<tr>
<td>≥</td>
<td>is greater than or equal to</td>
</tr>
<tr>
<td>∞</td>
<td>infinity</td>
</tr>
<tr>
<td>⇒</td>
<td>implies</td>
</tr>
<tr>
<td>⇔</td>
<td>is implied by</td>
</tr>
<tr>
<td>⇐</td>
<td>implies and is implied by (is equivalent to)</td>
</tr>
</tbody>
</table>
3 Operations

\[ a + b \quad \text{a plus } b \]
\[ a - b \quad \text{a minus } b \]
\[ a \times b, \quad ab \quad \text{a multiplied by } b \]
\[ a \div b, \quad \frac{a}{b} \quad \text{a divided by } b \]
\[ \sum_{i=1}^{n} a_i \quad a_1 + a_2 + \ldots + a_n \]
\[ \sqrt{a} \quad \text{the non-negative square root of } a, \text{ for } a \in \mathbb{R}, \ a \geq 0 \]
\[ \sqrt[n]{a} \quad \text{the (real) } n\text{th root of } a, \text{ for } a \in \mathbb{R}, \text{ where } \sqrt[n]{a} \geq 0 \text{ for } a \geq 0 \]
\[ |a| \quad \text{the modulus of } a \]
\[ n! \quad n \text{ factorial} \]
\[ \binom{n}{r} \quad \text{the binomial coefficient } \frac{n!}{r!(n-r)!} \text{ for } n, r \in \mathbb{Z} \text{ and } 0 \leq r \leq n \]

4 Functions

\[ f(x) \quad \text{the value of the function } f \text{ at } x \]
\[ f : A \to B \quad f \text{ is a function under which each element of set } A \text{ has an image in set } B \]
\[ f : x \mapsto y \quad \text{the function } f \text{ maps the element } x \text{ to the element } y \]
\[ f^{-1} \quad \text{the inverse function of the one-one function } f \]
\[ g \circ f \quad \text{the composite function of } f \text{ and } g \text{ which is defined by } g(f(x)) = g(f(x)) \]
\[ \lim_{x \to a} f(x) \quad \text{the limit of } f(x) \text{ as } x \text{ tends to } a \]
\[ \Delta x, \delta x \quad \text{an increment of } x \]
\[ \frac{dy}{dx} \quad \text{the derivative of } y \text{ with respect to } x \]
\[ \frac{d^n y}{dx^n} \quad \text{the } n\text{th derivative of } y \text{ with respect to } x \]
\[ f'(x), \ f''(x), \ldots, \ f^{(n)}(x) \quad \text{the first, second, } \ldots, \text{ } n\text{th derivatives of } f(x) \text{ with respect to } x \]
\[ \int y \, dx \quad \text{the indefinite integral of } y \text{ with respect to } x \]
\[ \int_a^b y \, dx \quad \text{the definite integral of } y \text{ with respect to } x \text{ between the limits } x = a \text{ and } x = b \]
\[ x, \dot{x}, \ldots \quad \text{the first, second, } \ldots \text{ derivatives of } x \text{ with respect to } t \]

5 Exponential and logarithmic functions

\[ e \quad \text{base of natural logarithms} \]
\[ e^x, \exp(x) \quad \text{exponential function of } x \]
\[ \log_a x \quad \text{logarithm to the base } a \text{ of } x \]
\[ \ln x \quad \text{natural logarithm of } x \]
\[ \log x, \log_{10} x \quad \text{logarithm of } x \text{ to base } 10 \]
6 Circular and hyperbolic functions

\begin{align*}
\sin, \cos, \tan \quad &\text{the circular functions} \\
\sec, \csc, \cot &\text{the inverse circular functions} \\
\sin^{-1}, \cos^{-1}, \tan^{-1} &\text{the hyperbolic functions} \\
\cosec, \sec, \cot &\text{the inverse hyperbolic functions} \\
\end{align*}

7 Complex numbers

\begin{align*}
i &\text{the imaginary unit, } i^2 = -1 \\
z &\text{a complex number, } z = x + iy = r(\cos \theta + i \sin \theta) \\
\text{Re } z &\text{the real part of } z, \text{Re } z = x \\
\text{Im } z &\text{the imaginary part of } z, \text{Im } z = y \\
|z| &\text{the modulus of } z, |z| = \sqrt{x^2 + y^2} \\
\text{arg } z &\text{the argument of } z, \text{arg } z = \theta \text{ where } -\pi < \theta \leq \pi \\
z^* &\text{the complex conjugate of } z, x - iy
\end{align*}

8 Matrices

\begin{align*}
M &\text{a matrix } M \\
M^{-1} &\text{the inverse of the non-singular square matrix } M \\
\text{det } M, |M| &\text{the determinant of the square matrix } M \\
I &\text{an identity (or unit) matrix}
\end{align*}

9 Vectors

\begin{align*}
a &\text{the vector } a \\
\overrightarrow{AB} &\text{the vector represented in magnitude and direction by the directed line segment } AB \\
\hat{a} &\text{a unit vector in the direction of } a \\
i, j, k &\text{unit vectors in the directions of the Cartesian coordinate axes} \\
\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix} &\text{the vectors } xi + yj \text{ (in 2 dimensions) and } xi + yj + zk \text{ (in 3 dimensions)} \\
|a|, a &\text{the magnitude of } a \\
\overrightarrow{AB}, |\overrightarrow{AB}| &\text{the magnitude of } \overrightarrow{AB} \\
a \cdot b &\text{the scalar product of } a \text{ and } b \\
a \times b &\text{the vector product of } a \text{ and } b
\end{align*}
10 Probability and statistics

\[ A, B, C, \ldots \] events

\[ A \cup B \] union of the events \( A \) and \( B \)

\[ A \cap B \] intersection of the events \( A \) and \( B \)

\[ P(A) \] probability of the event \( A \)

\[ A' \] complement of the event \( A \)

\[ P(A | B) \] probability of the event \( A \) conditional on the event \( B \)

\[ ^a C_r \] the number of combinations of \( r \) objects from \( n \), 
\[ ^a C_r = \frac{n!}{r!(n-r)!} \]

\[ ^a P_r \] the number of permutations of \( r \) objects from \( n \), 
\[ ^a P_r = \frac{n!}{(n-r)!} \]

\[ X, Y, R, \ldots \] random variables

\[ x, y, r, \ldots \] values of the random variables \( X, Y, R, \ldots \)

\[ x_1, x_2, \ldots \] observations

\[ f_1, f_2, \ldots \] frequencies with which the observations \( x_1, x_2, \ldots \) occur

\( p(x) \) probability function \( P(X = x) \) of the discrete random variable \( X \)

\( p_1, p_2, \ldots \) probabilities of the values \( x_1, x_2, \ldots \) of the discrete random variable \( X \)

\( f(x) \) value of the probability density function of a continuous random variable \( X \)

\( F(x) \) value of the cumulative distribution function of a continuous random variable \( X \)

\( E(X) \) expectation of the random variable \( X \)

\( E(g(X)) \) expectation of \( g(X) \)

\( \text{Var}(X) \) variance of the random variable \( X \)

\( G_t(t) \) probability generating function for the discrete random variable \( X \)

\( M_t(t) \) moment generating function for the continuous random variable \( X \)

\( B(n, p) \) binomial distribution with parameters \( n \) and \( p \)

\( \text{Geo}(p) \) geometric distribution with parameter \( p \)

\( \text{Po}(\lambda) \) Poisson distribution with parameter \( \lambda \)

\( N(\mu, \sigma^2) \) normal distribution with mean \( \mu \) and variance \( \sigma^2 \)

\( \mu \) population mean

\( \sigma^2 \) population variance

\( \sigma \) population standard deviation

\( \bar{x} \) sample mean, 
\[ \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \]

\( s^2 \) unbiased estimate of population variance from a sample, 
\[ s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \]

\( \rho \) product moment correlation coefficient for a population

\( r \) product moment correlation coefficient for a sample

\( \phi \) probability density function of the standardised normal variable \( Z \sim \text{N}(0, 1) \)

\( \Phi \) cumulative distribution function of the standardised normal variable \( Z \sim \text{N}(0, 1) \)

\( H_0, H_1 \) null and alternative hypotheses for a hypothesis test