



**Cambridge Assessment**  
International Education

# **Notation List**

**For Cambridge International Mathematics  
Qualifications**

**For use from 2020**

## Mathematical notation

Examinations for CIE syllabuses may use relevant notation from the following list.

### 1 Set notation

|                       |   |
|-----------------------|---|
| $\in$                 | is an element of  |
| $\notin$              | is not an element of  |
| $\{x_1, x_2, \dots\}$ | the set with elements $x_1, x_2, \dots$   |
| $\{x : \dots\}$       | the set of all $x$ such that ...  |
| $n(A)$                | the number of elements in set $A$   |
| $\emptyset$           | the empty set   |
| $\mathcal{E}$         | the universal set   |
| $\cup$                | the universal set (for 0607 IGCSE International Mathematics)                              |
| $A'$                  | the complement of the set $A$   |
| $\mathbb{N}$          | the set of natural numbers, $\{1, 2, 3, \dots\}$  |
| $\mathbb{Z}$          | the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$                                  |
| $\mathbb{Q}$          | the set of rational numbers, $\left\{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\right\}$ |
| $\mathbb{R}$          | the set of real numbers   |
| $\mathbb{C}$          | the set of complex numbers  |
| $(x, y)$              | the ordered pair $x, y$   |
| $\subseteq$           | is a subset of  |
| $\subset$             | is a proper subset of   |
| $\cup$                | union   |
| $\cap$                | intersection  |
| $[a, b]$              | the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$                              |
| $[a, b)$              | the interval $\{x \in \mathbb{R} : a \leq x < b\}$  |
| $(a, b]$              | the interval $\{x \in \mathbb{R} : a < x \leq b\}$  |
| $(a, b)$              | the open interval $\{x \in \mathbb{R} : a < x < b\}$                                      |
| $(S, \circ)$          | the group consisting of the set $S$ with binary operation $\circ$                         |

### 2 Miscellaneous symbols

|                   |  |
|-------------------|--|
| $=$               | is equal to                                  |
| $\neq$            | is not equal to                              |
| $\equiv$          | is identical to or is congruent to           |
| $\approx$         | is approximately equal to                    |
| $\sim$            | is distributed as                            |
| $\cong$           | is isomorphic to                             |
| $\propto$         | is proportional to                           |
| $<$               | is less than                                 |
| $\leq$            | is less than or equal to                     |
| $>$               | is greater than                              |
| $\geq$            | is greater than or equal to                  |
| $\infty$          | infinity                                     |
| $\Rightarrow$     | implies                                      |
| $\Leftarrow$      | is implied by                                |
| $\Leftrightarrow$ | implies and is implied by (is equivalent to) |

### 3 Operations

|                         |  |
|-------------------------|--|
| $a + b$                 | $a$ plus $b$   |
| $a - b$                 | $a$ minus $b$  |
| $a \times b, ab$        | $a$ multiplied by $b$  |
| $a \div b, \frac{a}{b}$ | $a$ divided by $b$   |
| $\sum_{i=1}^n a_i$      | $a_1 + a_2 + \dots + a_n$  |
| $\sqrt{a}$              | the non-negative square root of $a$ , for $a \in \mathbb{R}, a \geq 0$                             |
| $\sqrt[n]{a}$           | the (real) $n$ th root of $a$ , for $a \in \mathbb{R}$ , where $\sqrt[n]{a} \geq 0$ for $a \geq 0$ |
| $ a $                   | the modulus of $a$   |
| $n!$                    | $n$ factorial  |
| $\binom{n}{r}$          | the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}$ and $0 \leq r \leq n$     |

### 4 Functions

|                                    |   |
|------------------------------------|---|
| $f(x)$                             | the value of the function $f$ at $x$  |
| $f: A \rightarrow B$               | $f$ is a function under which each element of set $A$ has an image in set $B$           |
| $f: x \mapsto y$                   | the function $f$ maps the element $x$ to the element $y$                                |
| $f^{-1}$                           | the inverse function of the one-one function $f$  |
| $gf$                               | the composite function of $f$ and $g$ which is defined by $gf(x) = g(f(x))$             |
| $\lim_{x \rightarrow a} f(x)$      | the limit of $f(x)$ as $x$ tends to $a$   |
| $\Delta x, \delta x$               | an increment of $x$   |
| $\frac{dy}{dx}$                    | the derivative of $y$ with respect to $x$   |
| $\frac{d^n y}{dx^n}$               | the $n$ th derivative of $y$ with respect to $x$  |
| $f'(x), f''(x), \dots, f^{(n)}(x)$ | the first, second, $\dots$ , $n$ th derivatives of $f(x)$ with respect to $x$           |
| $\int y \, dx$                     | the indefinite integral of $y$ with respect to $x$                                      |
| $\int_a^b y \, dx$                 | the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$ |
| $\dot{x}, \ddot{x}, \dots$         | the first, second, $\dots$ derivatives of $x$ with respect to $t$                       |

### 5 Exponential and logarithmic functions

|                      |                                  |
|----------------------|----------------------------------|
| $e$                  | base of natural logarithms       |
| $e^x, \exp(x)$       | exponential function of $x$      |
| $\log_a x$           | logarithm to the base $a$ of $x$ |
| $\ln x$              | natural logarithm of $x$         |
| $\lg x, \log_{10} x$ | logarithm of $x$ to base 10      |

## 6 Circular and hyperbolic functions

|   |                                  |
|---|----------------------------------|
| $\left. \begin{array}{l} \sin, \cos, \tan \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$   | the circular functions           |
| $\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \end{array} \right\}$                                   | the inverse circular functions   |
| $\left. \begin{array}{l} \sinh, \cosh, \tanh \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$                               | the hyperbolic functions         |
| $\left. \begin{array}{l} \sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1} \end{array} \right\}$ | the inverse hyperbolic functions |

## 7 Complex numbers

|                       |  |
|-----------------------|--|
| $i$                   | the imaginary unit, $i^2 = -1$   |
| $z$                   | a complex number, $z = x + iy = r(\cos \theta + i \sin \theta)$        |
| $\operatorname{Re} z$ | the real part of $z$ , $\operatorname{Re} z = x$                       |
| $\operatorname{Im} z$ | the imaginary part of $z$ , $\operatorname{Im} z = y$                  |
| $ z $                 | the modulus of $z$ , $ z  = \sqrt{x^2 + y^2}$                          |
| $\arg z$              | the argument of $z$ , $\arg z = \theta$ where $-\pi < \theta \leq \pi$ |
| $z^*$                 | the complex conjugate of $z$ , $x - iy$                                |

## 8 Matrices

|                                 |  |
|---------------------------------|--|
| $\mathbf{M}$                    | a matrix $\mathbf{M}$                                      |
| $\mathbf{M}^{-1}$               | the inverse of the non-singular square matrix $\mathbf{M}$ |
| $\det \mathbf{M},  \mathbf{M} $ | the determinant of the square matrix $\mathbf{M}$          |
| $\mathbf{I}$                    | an identity (or unit) matrix                               |

## 9 Vectors

|   |   |
|---|---|
| $\mathbf{a}$  | the vector $\mathbf{a}$   |
| $\overline{AB}$   | the vector represented in magnitude and direction by the directed line segment $AB$                                       |
| $\hat{\mathbf{a}}$  | a unit vector in the direction of $\mathbf{a}$  |
| $\mathbf{i}, \mathbf{j}, \mathbf{k}$  | unit vectors in the directions of the Cartesian coordinate axes   |
| $\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ | the vectors $x\mathbf{i} + y\mathbf{j}$ (in 2 dimensions) and $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ (in 3 dimensions) |
| $ \mathbf{a} , a$   | the magnitude of $\mathbf{a}$   |
| $ \overline{AB} , AB$   | the magnitude of $\overline{AB}$  |
| $\mathbf{a} \cdot \mathbf{b}$   | the scalar product of $\mathbf{a}$ and $\mathbf{b}$   |
| $\mathbf{a} \times \mathbf{b}$  | the vector product of $\mathbf{a}$ and $\mathbf{b}$   |

## 10 Probability and statistics

|                      |  |
|----------------------|--|
| $A, B, C, \dots$     | events   |
| $A \cup B$           | union of the events $A$ and $B$  |
| $A \cap B$           | intersection of the events $A$ and $B$   |
| $P(A)$               | probability of the event $A$   |
| $A'$                 | complement of the event $A$  |
| $P(A   B)$           | probability of the event $A$ conditional on the event $B$  |
| ${}^n C_r$           | the number of combinations of $r$ objects from $n$ , ${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$         |
| ${}^n P_r$           | the number of permutations of $r$ objects from $n$ , ${}^n P_r = \frac{n!}{(n-r)!}$                          |
| $X, Y, R, \dots$     | random variables   |
| $x, y, r, \dots$     | values of the random variables $X, Y, R, \dots$  |
| $x_1, x_2, \dots$    | observations   |
| $f_1, f_2, \dots$    | frequencies with which the observations $x_1, x_2, \dots$ occur  |
| $p(x)$               | probability function $P(X = x)$ of the discrete random variable $X$  |
| $p_1, p_2, \dots$    | probabilities of the values $x_1, x_2, \dots$ of the discrete random variable $X$                            |
| $f(x)$               | value of the probability density function of a continuous random variable $X$                                |
| $F(x)$               | value of the cumulative distribution function of a continuous random variable $X$                            |
| $E(X)$               | expectation of the random variable $X$   |
| $E(g(X))$            | expectation of $g(X)$  |
| $\text{Var}(X)$      | variance of the random variable $X$  |
| $G_X(t)$             | probability generating function for the discrete random variable $X$   |
| $M_X(t)$             | moment generating function for the random variable $X$   |
| $B(n, p)$            | binomial distribution with parameters $n$ and $p$  |
| $\text{Geo}(p)$      | geometric distribution with parameter $p$  |
| $\text{Po}(\lambda)$ | Poisson distribution with parameter $\lambda$  |
| $N(\mu, \sigma^2)$   | normal distribution with mean $\mu$ and variance $\sigma^2$  |
| $\mu$                | population mean  |
| $\sigma^2$           | population variance  |
| $\sigma$             | population standard deviation  |
| $\bar{x}$            | sample mean, $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  |
| $s^2$                | unbiased estimate of population variance from a sample, $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$ |
| $\rho$               | product moment correlation coefficient for a population  |
| $r$                  | product moment correlation coefficient for a sample  |
| $\phi$               | probability density function of the standardised normal variable $Z \sim N(0, 1)$                            |
| $\Phi$               | cumulative distribution function of the standardised normal variable $Z \sim N(0, 1)$                        |
| $H_0, H_1$           | null and alternative hypotheses for a hypothesis test  |