

# List MF20

## List of Formulae and Statistical Tables

**Cambridge Pre-U Mathematics (9794) and  
Further Mathematics (9795)**

**For use from 2017 in all papers for the above syllabuses.**

CST317



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**CAMBRIDGE**  
International Examinations

## PURE MATHEMATICS

### Mensuration

$$\text{Surface area of sphere} = 4\pi r^2$$

$$\text{Area of curved surface of cone} = \pi r \times \text{slant height}$$

### Trigonometry

$$a^2 = b^2 + c^2 - 2bc \cos A$$

### Arithmetic series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2} n(a + l) = \frac{1}{2} n\{2a + (n - 1)d\}$$

### Geometric series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

### Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6} n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4} n^2(n+1)^2$$

### Binomial series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n, (n \in \mathbb{N}), \text{ where } \binom{n}{r} = {}^n C_r = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2} x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r} x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

### Logarithms and exponentials

$$e^{x \ln a} = a^x$$

### Complex numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n(\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of  $z^n = 1$  are given by  $z = e^{\frac{2\pi k i}{n}}$ , for  $k = 0, 1, 2, \dots, n-1$

## Maclaurin's series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

## Hyperbolic functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln \{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln \{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad (|x| < 1)$$

## Coordinate geometry

The perpendicular distance from  $(h, k)$  to  $ax + by + c = 0$  is  $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients  $m_1$  and  $m_2$  is  $\tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

### Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\text{For } t = \tan \frac{1}{2}A : \sin A = \frac{2t}{1+t^2}, \cos A = \frac{1-t^2}{1+t^2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

### Vectors

The resolved part of  $\mathbf{a}$  in the direction of  $\mathbf{b}$  is  $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing  $AB$  in the ratio  $\lambda : \mu$  is  $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

$$\text{Vector product: } \mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

If  $A$  is the point with position vector  $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$  and the direction vector  $\mathbf{b}$  is given by

$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ , then the straight line through  $A$  with direction vector  $\mathbf{b}$  has cartesian equation

$$\frac{x-a_1}{b_1} = \frac{y-a_2}{b_2} = \frac{z-a_3}{b_3} (= \lambda)$$

The plane through  $A$  with normal vector  $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$  has cartesian equation  $n_1x + n_2y + n_3z + d = 0$   
where  $d = -\mathbf{a} \cdot \mathbf{n}$

The plane through non-collinear points  $A, B$  and  $C$  has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector  $\mathbf{a}$  and parallel to  $\mathbf{b}$  and  $\mathbf{c}$  has equation  $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of  $(\alpha, \beta, \gamma)$  from  $n_1x + n_2y + n_3z + d = 0$  is  $\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

### Matrix transformations

Anticlockwise rotation through  $\theta$  about  $O$ :  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line  $y = (\tan \theta)x$ :  $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

## Differentiation

| $f(x)$                   | $f'(x)$                          |
|--------------------------|----------------------------------|
| $\tan kx$                | $k \sec^2 kx$                    |
| $\sin^{-1} x$            | $\frac{1}{\sqrt{1-x^2}}$         |
| $\cos^{-1} x$            | $-\frac{1}{\sqrt{1-x^2}}$        |
| $\tan^{-1} x$            | $\frac{1}{1+x^2}$                |
| $\sec x$                 | $\sec x \tan x$                  |
| $\cot x$                 | $-\operatorname{cosec}^2 x$      |
| $\operatorname{cosec} x$ | $-\operatorname{cosec} x \cot x$ |
| $\sinh x$                | $\cosh x$                        |
| $\cosh x$                | $\sinh x$                        |
| $\tanh x$                | $\operatorname{sech}^2 x$        |
| $\sinh^{-1} x$           | $\frac{1}{\sqrt{1+x^2}}$         |
| $\cosh^{-1} x$           | $\frac{1}{\sqrt{x^2-1}}$         |
| $\tanh^{-1} x$           | $\frac{1}{1-x^2}$                |

## Integration (+ constant; $a > 0$ where relevant)

| $f(x)$                       | $\int f(x) dx$  |
|------------------------------|---|
| $\sec^2 kx$                  | $\frac{1}{k} \tan kx$   |
| $\tan x$                     | $\ln \sec x $   |
| $\cot x$                     | $\ln \sin x $   |
| $\operatorname{cosec} x$     | $-\ln \operatorname{cosec} x + \cot x  = \ln \tan(\frac{1}{2}x) $   |
| $\sec x$                     | $\ln \sec x + \tan x  = \ln \tan(\frac{1}{2}x + \frac{1}{4}\pi) $   |
| $\sinh x$                    | $\cosh x$   |
| $\cosh x$                    | $\sinh x$   |
| $\tanh x$                    | $\ln \cosh x$   |
| $\frac{1}{\sqrt{a^2 - x^2}}$ | $\sin^{-1}\left(\frac{x}{a}\right)$ ( $ x  < a$ )   |
| $\frac{1}{a^2 + x^2}$        | $\frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$   |
| $\frac{1}{\sqrt{x^2 - a^2}}$ | $\cosh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 - a^2}\}$ ( $x > a$ )                             |
| $\frac{1}{\sqrt{a^2 + x^2}}$ | $\sinh^{-1}\left(\frac{x}{a}\right)$ or $\ln\{x + \sqrt{x^2 + a^2}\}$   |
| $\frac{1}{a^2 - x^2}$        | $\frac{1}{2a} \ln\left \frac{a+x}{a-x}\right  = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right)$ ( $ x  < a$ ) |
| $\frac{1}{x^2 - a^2}$        | $\frac{1}{2a} \ln\left \frac{x-a}{x+a}\right $  |
| $\int u \frac{dv}{dx} dx$    | $= uv - \int v \frac{du}{dx} dx$  |

**Area of a sector**

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

$$A = \frac{1}{2} \int \left( x \frac{dy}{dt} - y \frac{dx}{dt} \right) dt \quad (\text{parametric form})$$

**Arc length**

$$s = \int \sqrt{1 + \left( \frac{dy}{dx} \right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2} dt \quad (\text{parametric form})$$

$$s = \int \sqrt{r^2 + \left( \frac{dr}{d\theta} \right)^2} d\theta \quad (\text{polar form})$$

**Surface area of revolution**

$$S_x = 2\pi \int y ds$$

$$S_y = 2\pi \int x ds$$

**Numerical solution of equations**

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

## MECHANICS

### Motion in a circle

Transverse velocity:  $v = r\dot{\theta}$

Transverse acceleration:  $\dot{v} = r\ddot{\theta}$

Radial acceleration:  $-r\dot{\theta}^2 = -\frac{v^2}{r}$

## PROBABILITY

### Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) P(B | A)$$

$$P(A | B) = \frac{P(B | A) P(A)}{P(B | A) P(A) + P(B | A') P(A')}$$

$$\text{Bayes' Theorem: } P(A_j | B) = \frac{P(A_j) P(B | A_j)}{\sum P(A_i) P(B | A_i)}$$

### Discrete distributions

For a discrete random variable  $X$  taking values  $x_i$  with probabilities  $p_i$

Expectation (mean):  $E(X) = \mu = \sum x_i p_i$

Variance:  $\text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$

For a function  $g(X)$ :  $E(g(X)) = \sum g(x_i) p_i$

The probability generating function (P.G.F.) of  $X$  is  $G_X(t) = E(t^X)$ ,

and  $E(X) = G'_X(1)$ ,  $\text{Var}(X) = G''_X(1) + G'_X(1) - \{G'_X(1)\}^2$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $G_Z(t) = G_X(t) G_Y(t)$

The moment generating function (M.G.F.) of  $X$  is  $M_X(t) = E(e^{tX})$ ,

and  $E(X) = M'_X(0)$ ,  $E(X^n) = M_X^{(n)}(0)$ ,  $\text{Var}(X) = M''_X(0) - \{M'_X(0)\}^2$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $M_Z(t) = M_X(t) M_Y(t)$

### Standard discrete distributions

| Distribution of $X$                    | $P(X = x)$                          | Mean          | Variance          | P.G.F.                | M.G.F.                    |
|--|-------------------------------------|---------------|-------------------|-----------------------|---------------------------|
| Binomial $B(n, p)$                     | $\binom{n}{x} p^x (1-p)^{n-x}$      | $np$          | $np(1-p)$         | $(1-p+pt)^n$          | $(1-p+pe^t)^n$            |
| Poisson $\text{Po}(\lambda)$           | $e^{-\lambda} \frac{\lambda^x}{x!}$ | $\lambda$     | $\lambda$         | $e^{\lambda(t-1)}$    | $e^{\lambda(e^t-1)}$      |
| Geometric $\text{Geo}(p)$ on 1, 2, ... | $p(1-p)^{x-1}$                      | $\frac{1}{p}$ | $\frac{1-p}{p^2}$ | $\frac{pt}{1-(1-p)t}$ | $\frac{pe^t}{1-(1-p)e^t}$ |

## Continuous distributions

For a continuous random variable  $X$  having probability density function (P.D.F.) f

$$\text{Expectation (mean): } E(X) = \mu = \int x f(x) dx$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{For a function } g(X) : E(g(X)) = \int g(x) f(x) dx$$

$$\text{Cumulative distribution function: } F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

The moment generating function (M.G.F.) of  $X$  is  $M_X(t) = E(e^{tX})$ ,

$$\text{and } E(X) = M'_X(0), E(X^n) = M_X^{(n)}(0), \text{Var}(X) = M''_X(0) - \{M'_X(0)\}^2$$

For  $Z = X + Y$ , where  $X$  and  $Y$  are independent:  $M_Z(t) = M_X(t) M_Y(t)$

## Standard continuous distributions

| Distribution of $X$               | P.D.F   | Mean                | Variance              | M.G.F.                                |
|-----------------------------------|---|---------------------|-----------------------|---------------------------------------|
| Uniform (Rectangular) on $[a, b]$ | $\frac{1}{b-a}$   | $\frac{1}{2}(a+b)$  | $\frac{1}{12}(b-a)^2$ | $\frac{e^{bt} - e^{at}}{(b-a)t}$      |
| Exponential                       | $\lambda e^{-\lambda x}$  | $\frac{1}{\lambda}$ | $\frac{1}{\lambda^2}$ | $\frac{\lambda}{\lambda-t}$           |
| Normal $N(\mu, \sigma^2)$         | $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ | $\mu$               | $\sigma^2$            | $e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ |

## Expectation algebra

For independent random variables  $X$  and  $Y$

$$E(XY) = E(X) E(Y), \text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

## Sampling distributions

For a random sample  $X_1, X_2, \dots, X_n$  of  $n$  independent observations from a distribution having mean  $\mu$  and variance  $\sigma^2$

$\bar{X}$  is an unbiased estimator of  $\mu$ , with  $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$

$S^2$  is an unbiased estimator of  $\sigma^2$ , where  $S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$

For a random sample of  $n$  observations from  $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma / \sqrt{n}} \sim N(0,1)$$

$$\frac{\bar{X} - \mu}{S / \sqrt{n}} \sim t_{n-1} \text{ (also valid in matched-pairs situations)}$$

If  $X$  is the observed number of successes in  $n$  independent Bernoulli trials, in each of which the probability of success is  $p$ , and  $Y = \frac{X}{n}$ , then  $E(Y) = p$  and  $\text{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of  $n_x$  observations from  $N(\mu_x, \sigma_x^2)$  and, independently, a random sample of  $n_y$  observations from  $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

If  $\sigma_x^2 = \sigma_y^2 = \sigma^2$  (unknown) then  $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left( \frac{1}{n_x} + \frac{1}{n_y} \right)}} \sim t_{n_x+n_y-2}$ , where  $S_p^2 = \frac{(n_x-1)S_x^2 + (n_y-1)S_y^2}{n_x+n_y-2}$

## Correlation and Regression

For a set of  $n$  pairs of values  $(x_i, y_i)$

$$S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

$$S_{yy} = \sum(y_i - \bar{y})^2 = \sum y_i^2 - \frac{(\sum y_i)^2}{n}$$

$$S_{xy} = \sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2}} = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sqrt{\left( \sum x_i^2 - \frac{(\sum x_i)^2}{n} \right) \left( \sum y_i^2 - \frac{(\sum y_i)^2}{n} \right)}}$$

The regression coefficient of  $y$  on  $x$  is  $b = \frac{S_{xy}}{S_{xx}} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$

Least squares regression line of  $y$  on  $x$  is  $y = a + bx$  where  $a = \bar{y} - b\bar{x}$













### CUMULATIVE POISSON PROBABILITIES

$$P(X \leq x) = \sum_{r=0}^x e^{-\lambda} \frac{\lambda^r}{r!}$$

| $\lambda$ | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$   | 0.9900 | 0.9802 | 0.9704 | 0.9608 | 0.9512 | 0.9418 | 0.9324 | 0.9231 | 0.9139 |
| 1         | 1.0000 | 0.9998 | 0.9996 | 0.9992 | 0.9988 | 0.9983 | 0.9977 | 0.9970 | 0.9962 |
| 2         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 |
| 3         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\lambda$ | 0.10   | 0.20   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| $x = 0$   | 0.9048 | 0.8187 | 0.7408 | 0.6703 | 0.6065 | 0.5488 | 0.4966 | 0.4493 | 0.4066 |
| 1         | 0.9953 | 0.9825 | 0.9631 | 0.9384 | 0.9098 | 0.8781 | 0.8442 | 0.8088 | 0.7725 |
| 2         | 0.9998 | 0.9989 | 0.9964 | 0.9921 | 0.9856 | 0.9769 | 0.9659 | 0.9526 | 0.9371 |
| 3         | 1.0000 | 0.9999 | 0.9997 | 0.9992 | 0.9982 | 0.9966 | 0.9942 | 0.9909 | 0.9865 |
| 4         | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9986 | 0.9977 |
| 5         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9997 |
| 6         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\lambda$ | 1.00   | 1.10   | 1.20   | 1.30   | 1.40   | 1.50   | 1.60   | 1.70   | 1.80   |
| $x = 0$   | 0.3679 | 0.3329 | 0.3012 | 0.2725 | 0.2466 | 0.2231 | 0.2019 | 0.1827 | 0.1653 |
| 1         | 0.7358 | 0.6990 | 0.6626 | 0.6268 | 0.5918 | 0.5578 | 0.5249 | 0.4932 | 0.4628 |
| 2         | 0.9197 | 0.9004 | 0.8795 | 0.8571 | 0.8335 | 0.8088 | 0.7834 | 0.7572 | 0.7306 |
| 3         | 0.9810 | 0.9743 | 0.9662 | 0.9569 | 0.9463 | 0.9344 | 0.9212 | 0.9068 | 0.8913 |
| 4         | 0.9963 | 0.9946 | 0.9923 | 0.9893 | 0.9857 | 0.9814 | 0.9763 | 0.9704 | 0.9636 |
| 5         | 0.9994 | 0.9990 | 0.9985 | 0.9978 | 0.9968 | 0.9955 | 0.9940 | 0.9920 | 0.9896 |
| 6         | 0.9999 | 0.9999 | 0.9997 | 0.9996 | 0.9994 | 0.9991 | 0.9987 | 0.9981 | 0.9974 |
| 7         | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9994 |
| 8         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 9         | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\lambda$ | 2.00   | 2.10   | 2.20   | 2.30   | 2.40   | 2.50   | 2.60   | 2.70   | 2.80   |
| $x = 0$   | 0.1353 | 0.1225 | 0.1108 | 0.1003 | 0.0907 | 0.0821 | 0.0743 | 0.0672 | 0.0608 |
| 1         | 0.4060 | 0.3796 | 0.3546 | 0.3309 | 0.3084 | 0.2873 | 0.2674 | 0.2487 | 0.2311 |
| 2         | 0.6767 | 0.6496 | 0.6227 | 0.5960 | 0.5697 | 0.5438 | 0.5184 | 0.4936 | 0.4695 |
| 3         | 0.8571 | 0.8386 | 0.8194 | 0.7993 | 0.7787 | 0.7576 | 0.7360 | 0.7141 | 0.6919 |
| 4         | 0.9473 | 0.9379 | 0.9275 | 0.9162 | 0.9041 | 0.8912 | 0.8774 | 0.8629 | 0.8477 |
| 5         | 0.9834 | 0.9796 | 0.9751 | 0.9700 | 0.9643 | 0.9580 | 0.9510 | 0.9433 | 0.9349 |
| 6         | 0.9955 | 0.9941 | 0.9925 | 0.9906 | 0.9884 | 0.9858 | 0.9828 | 0.9794 | 0.9756 |
| 7         | 0.9989 | 0.9985 | 0.9980 | 0.9974 | 0.9967 | 0.9958 | 0.9947 | 0.9934 | 0.9919 |
| 8         | 0.9998 | 0.9997 | 0.9995 | 0.9994 | 0.9991 | 0.9989 | 0.9985 | 0.9981 | 0.9976 |
| 9         | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9993 |
| 10        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 |
| 11        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 12        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| $\lambda$ | 3.00   | 3.10   | 3.20   | 3.30   | 3.40   | 3.50   | 3.60   | 3.70   | 3.80   |
| $x = 0$   | 0.0498 | 0.0450 | 0.0408 | 0.0369 | 0.0334 | 0.0302 | 0.0273 | 0.0247 | 0.0224 |
| 1         | 0.1991 | 0.1847 | 0.1712 | 0.1586 | 0.1468 | 0.1359 | 0.1257 | 0.1162 | 0.1074 |
| 2         | 0.4232 | 0.4012 | 0.3799 | 0.3594 | 0.3397 | 0.3208 | 0.3027 | 0.2854 | 0.2689 |
| 3         | 0.6472 | 0.6248 | 0.6025 | 0.5803 | 0.5584 | 0.5366 | 0.5152 | 0.4942 | 0.4735 |
| 4         | 0.8153 | 0.7982 | 0.7806 | 0.7626 | 0.7442 | 0.7254 | 0.7064 | 0.6872 | 0.6678 |
| 5         | 0.9161 | 0.9057 | 0.8946 | 0.8829 | 0.8705 | 0.8576 | 0.8441 | 0.8301 | 0.8156 |
| 6         | 0.9665 | 0.9612 | 0.9554 | 0.9490 | 0.9421 | 0.9347 | 0.9267 | 0.9182 | 0.9091 |
| 7         | 0.9881 | 0.9858 | 0.9832 | 0.9802 | 0.9769 | 0.9733 | 0.9692 | 0.9648 | 0.9599 |
| 8         | 0.9962 | 0.9953 | 0.9943 | 0.9931 | 0.9917 | 0.9901 | 0.9883 | 0.9863 | 0.9840 |
| 9         | 0.9989 | 0.9986 | 0.9982 | 0.9978 | 0.9973 | 0.9967 | 0.9960 | 0.9952 | 0.9942 |
| 10        | 0.9997 | 0.9996 | 0.9995 | 0.9994 | 0.9992 | 0.9990 | 0.9987 | 0.9984 | 0.9981 |
| 11        | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9996 | 0.9995 | 0.9994 |
| 12        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 |
| 13        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 14        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

### CUMULATIVE POISSON PROBABILITIES

| $\lambda$ | 4.00   | 4.10   | 4.20   | 4.30   | 4.40   | 4.50   | 4.60   | 4.70   | 4.80   | 4.90   |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$   | 0.0183 | 0.0166 | 0.0150 | 0.0136 | 0.0123 | 0.0111 | 0.0101 | 0.0091 | 0.0082 | 0.0074 |
| 1         | 0.0916 | 0.0845 | 0.0780 | 0.0719 | 0.0663 | 0.0611 | 0.0563 | 0.0518 | 0.0477 | 0.0439 |
| 2         | 0.2381 | 0.2238 | 0.2102 | 0.1974 | 0.1851 | 0.1736 | 0.1626 | 0.1523 | 0.1425 | 0.1333 |
| 3         | 0.4335 | 0.4142 | 0.3954 | 0.3772 | 0.3594 | 0.3423 | 0.3257 | 0.3097 | 0.2942 | 0.2793 |
| 4         | 0.6288 | 0.6093 | 0.5898 | 0.5704 | 0.5512 | 0.5321 | 0.5132 | 0.4946 | 0.4763 | 0.4582 |
| 5         | 0.7851 | 0.7693 | 0.7531 | 0.7367 | 0.7199 | 0.7029 | 0.6858 | 0.6684 | 0.6510 | 0.6335 |
| 6         | 0.8893 | 0.8786 | 0.8675 | 0.8558 | 0.8436 | 0.8311 | 0.8180 | 0.8046 | 0.7908 | 0.7767 |
| 7         | 0.9489 | 0.9427 | 0.9361 | 0.9290 | 0.9214 | 0.9134 | 0.9049 | 0.8960 | 0.8867 | 0.8769 |
| 8         | 0.9786 | 0.9755 | 0.9721 | 0.9683 | 0.9642 | 0.9597 | 0.9549 | 0.9497 | 0.9442 | 0.9382 |
| 9         | 0.9919 | 0.9905 | 0.9889 | 0.9871 | 0.9851 | 0.9829 | 0.9805 | 0.9778 | 0.9749 | 0.9717 |
| 10        | 0.9972 | 0.9966 | 0.9959 | 0.9952 | 0.9943 | 0.9933 | 0.9922 | 0.9910 | 0.9896 | 0.9880 |
| 11        | 0.9991 | 0.9989 | 0.9986 | 0.9983 | 0.9980 | 0.9976 | 0.9971 | 0.9966 | 0.9960 | 0.9953 |
| 12        | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9993 | 0.9992 | 0.9990 | 0.9988 | 0.9986 | 0.9983 |
| 13        | 0.9999 | 0.9999 | 0.9999 | 0.9998 | 0.9998 | 0.9997 | 0.9997 | 0.9996 | 0.9995 | 0.9994 |
| 14        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 15        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 16        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

| $\lambda$ | 5.00   | 5.50   | 6.00   | 6.50   | 7.00   | 7.50   | 8.00   | 8.50   | 9.00   | 9.50   |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$   | 0.0067 | 0.0041 | 0.0025 | 0.0015 | 0.0009 | 0.0006 | 0.0003 | 0.0002 | 0.0001 | 0.0001 |
| 1         | 0.0404 | 0.0266 | 0.0174 | 0.0113 | 0.0073 | 0.0047 | 0.0030 | 0.0019 | 0.0012 | 0.0008 |
| 2         | 0.1247 | 0.0884 | 0.0620 | 0.0430 | 0.0296 | 0.0203 | 0.0138 | 0.0093 | 0.0062 | 0.0042 |
| 3         | 0.2650 | 0.2017 | 0.1512 | 0.1118 | 0.0818 | 0.0591 | 0.0424 | 0.0301 | 0.0212 | 0.0149 |
| 4         | 0.4405 | 0.3575 | 0.2851 | 0.2237 | 0.1730 | 0.1321 | 0.0996 | 0.0744 | 0.0550 | 0.0403 |
| 5         | 0.6160 | 0.5289 | 0.4457 | 0.3690 | 0.3007 | 0.2414 | 0.1912 | 0.1496 | 0.1157 | 0.0885 |
| 6         | 0.7622 | 0.6860 | 0.6063 | 0.5265 | 0.4497 | 0.3782 | 0.3134 | 0.2562 | 0.2068 | 0.1649 |
| 7         | 0.8666 | 0.8095 | 0.7440 | 0.6728 | 0.5987 | 0.5246 | 0.4530 | 0.3856 | 0.3239 | 0.2687 |
| 8         | 0.9319 | 0.8944 | 0.8472 | 0.7916 | 0.7291 | 0.6620 | 0.5925 | 0.5231 | 0.4557 | 0.3918 |
| 9         | 0.9682 | 0.9462 | 0.9161 | 0.8774 | 0.8305 | 0.7764 | 0.7166 | 0.6530 | 0.5874 | 0.5218 |
| 10        | 0.9863 | 0.9747 | 0.9574 | 0.9332 | 0.9015 | 0.8622 | 0.8159 | 0.7634 | 0.7060 | 0.6453 |
| 11        | 0.9945 | 0.9890 | 0.9799 | 0.9661 | 0.9467 | 0.9208 | 0.8881 | 0.8487 | 0.8030 | 0.7520 |
| 12        | 0.9980 | 0.9955 | 0.9912 | 0.9840 | 0.9730 | 0.9573 | 0.9362 | 0.9091 | 0.8758 | 0.8364 |
| 13        | 0.9993 | 0.9983 | 0.9964 | 0.9929 | 0.9872 | 0.9784 | 0.9658 | 0.9486 | 0.9261 | 0.8981 |
| 14        | 0.9998 | 0.9994 | 0.9986 | 0.9970 | 0.9943 | 0.9897 | 0.9827 | 0.9726 | 0.9585 | 0.9400 |
| 15        | 0.9999 | 0.9998 | 0.9995 | 0.9988 | 0.9976 | 0.9954 | 0.9918 | 0.9862 | 0.9780 | 0.9665 |
| 16        | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9990 | 0.9980 | 0.9963 | 0.9934 | 0.9889 | 0.9823 |
| 17        | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9992 | 0.9984 | 0.9970 | 0.9947 | 0.9911 |
| 18        | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 | 0.9997 | 0.9993 | 0.9987 | 0.9976 | 0.9957 |
| 19        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9995 | 0.9989 | 0.9980 |
| 20        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 | 0.9991 |
| 21        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9996 |        |
| 22        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9999 |        |
| 23        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |        |
| 24        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |        |

### CUMULATIVE POISSON PROBABILITIES

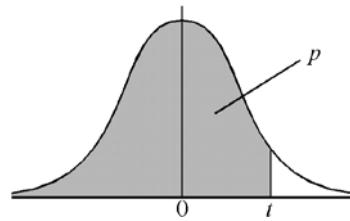
| $\lambda$ | 10.00  | 11.00  | 12.00  | 13.00  | 14.00  | 15.00  | 16.00  | 17.00  | 18.00  | 19.00  |
|-----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $x = 0$   | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 1         | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 2         | 0.0028 | 0.0012 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3         | 0.0103 | 0.0049 | 0.0023 | 0.0011 | 0.0005 | 0.0002 | 0.0001 | 0.0000 | 0.0000 | 0.0000 |
| 4         | 0.0293 | 0.0151 | 0.0076 | 0.0037 | 0.0018 | 0.0009 | 0.0004 | 0.0002 | 0.0001 | 0.0000 |
| 5         | 0.0671 | 0.0375 | 0.0203 | 0.0107 | 0.0055 | 0.0028 | 0.0014 | 0.0007 | 0.0003 | 0.0002 |
| 6         | 0.1301 | 0.0786 | 0.0458 | 0.0259 | 0.0142 | 0.0076 | 0.0040 | 0.0021 | 0.0010 | 0.0005 |
| 7         | 0.2202 | 0.1432 | 0.0895 | 0.0540 | 0.0316 | 0.0180 | 0.0100 | 0.0054 | 0.0029 | 0.0015 |
| 8         | 0.3328 | 0.2320 | 0.1550 | 0.0998 | 0.0621 | 0.0374 | 0.0220 | 0.0126 | 0.0071 | 0.0039 |
| 9         | 0.4579 | 0.3405 | 0.2424 | 0.1658 | 0.1094 | 0.0699 | 0.0433 | 0.0261 | 0.0154 | 0.0089 |
| 10        | 0.5830 | 0.4599 | 0.3472 | 0.2517 | 0.1757 | 0.1185 | 0.0774 | 0.0491 | 0.0304 | 0.0183 |
| 11        | 0.6968 | 0.5793 | 0.4616 | 0.3532 | 0.2600 | 0.1848 | 0.1270 | 0.0847 | 0.0549 | 0.0347 |
| 12        | 0.7916 | 0.6887 | 0.5760 | 0.4631 | 0.3585 | 0.2676 | 0.1931 | 0.1350 | 0.0917 | 0.0606 |
| 13        | 0.8645 | 0.7813 | 0.6815 | 0.5730 | 0.4644 | 0.3632 | 0.2745 | 0.2009 | 0.1426 | 0.0984 |
| 14        | 0.9165 | 0.8540 | 0.7720 | 0.6751 | 0.5704 | 0.4657 | 0.3675 | 0.2808 | 0.2081 | 0.1497 |
| 15        | 0.9513 | 0.9074 | 0.8444 | 0.7636 | 0.6694 | 0.5681 | 0.4667 | 0.3715 | 0.2867 | 0.2148 |
| 16        | 0.9730 | 0.9441 | 0.8987 | 0.8355 | 0.7559 | 0.6641 | 0.5660 | 0.4677 | 0.3751 | 0.2920 |
| 17        | 0.9857 | 0.9678 | 0.9370 | 0.8905 | 0.8272 | 0.7489 | 0.6593 | 0.5640 | 0.4686 | 0.3784 |
| 18        | 0.9928 | 0.9823 | 0.9626 | 0.9302 | 0.8826 | 0.8195 | 0.7423 | 0.6550 | 0.5622 | 0.4695 |
| 19        | 0.9965 | 0.9907 | 0.9787 | 0.9573 | 0.9235 | 0.8752 | 0.8122 | 0.7363 | 0.6509 | 0.5606 |
| 20        | 0.9984 | 0.9953 | 0.9884 | 0.9750 | 0.9521 | 0.9170 | 0.8682 | 0.8055 | 0.7307 | 0.6472 |
| 21        | 0.9993 | 0.9977 | 0.9939 | 0.9859 | 0.9712 | 0.9469 | 0.9108 | 0.8615 | 0.7991 | 0.7255 |
| 22        | 0.9997 | 0.9990 | 0.9970 | 0.9924 | 0.9833 | 0.9673 | 0.9418 | 0.9047 | 0.8551 | 0.7931 |
| 23        | 0.9999 | 0.9995 | 0.9985 | 0.9960 | 0.9907 | 0.9805 | 0.9633 | 0.9367 | 0.8989 | 0.8490 |
| 24        | 1.0000 | 0.9998 | 0.9993 | 0.9980 | 0.9950 | 0.9888 | 0.9777 | 0.9594 | 0.9317 | 0.8933 |
| 25        | 1.0000 | 0.9999 | 0.9997 | 0.9990 | 0.9974 | 0.9938 | 0.9869 | 0.9748 | 0.9554 | 0.9269 |
| 26        | 1.0000 | 1.0000 | 0.9999 | 0.9995 | 0.9987 | 0.9967 | 0.9925 | 0.9848 | 0.9718 | 0.9514 |
| 27        | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9983 | 0.9959 | 0.9912 | 0.9827 | 0.9687 |
| 28        | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9991 | 0.9978 | 0.9950 | 0.9897 | 0.9805 |
| 29        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9989 | 0.9973 | 0.9941 | 0.9882 |
| 30        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 | 0.9986 | 0.9967 | 0.9930 |
| 31        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 | 0.9993 | 0.9982 | 0.9960 |
| 32        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9996 | 0.9990 | 0.9978 |
| 33        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9995 | 0.9988 |
| 34        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 | 0.9994 |
| 35        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9997 |
| 36        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 | 0.9998 |
| 37        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 0.9999 |
| 38        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |



## CRITICAL VALUES FOR THE $t$ -DISTRIBUTION

If  $T$  has a  $t$ -distribution with  $v$  degrees of freedom, then, for each pair of values of  $p$  and  $v$ , the table gives the value of  $t$  such that:

$$P(T \leq t) = p.$$



| $p$      | 0.75  | 0.90  | 0.95  | 0.975 | 0.99  | 0.995 | 0.9975 | 0.999 | 0.9995 |
|----------|-------|-------|-------|-------|-------|-------|--------|-------|--------|
| $v=1$    | 1.000 | 3.078 | 6.314 | 12.71 | 31.82 | 63.66 | 127.3  | 318.3 | 636.6  |
| 2        | 0.816 | 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 14.09  | 22.33 | 31.60  |
| 3        | 0.765 | 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 7.453  | 10.21 | 12.92  |
| 4        | 0.741 | 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 5.598  | 7.173 | 8.610  |
| 5        | 0.727 | 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 4.773  | 5.894 | 6.869  |
| 6        | 0.718 | 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 4.317  | 5.208 | 5.959  |
| 7        | 0.711 | 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 4.029  | 4.785 | 5.408  |
| 8        | 0.706 | 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 3.833  | 4.501 | 5.041  |
| 9        | 0.703 | 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 3.690  | 4.297 | 4.781  |
| 10       | 0.700 | 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 3.581  | 4.144 | 4.587  |
| 11       | 0.697 | 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 3.497  | 4.025 | 4.437  |
| 12       | 0.695 | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 3.428  | 3.930 | 4.318  |
| 13       | 0.694 | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 3.372  | 3.852 | 4.221  |
| 14       | 0.692 | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 3.326  | 3.787 | 4.140  |
| 15       | 0.691 | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 3.286  | 3.733 | 4.073  |
| 16       | 0.690 | 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 3.252  | 3.686 | 4.015  |
| 17       | 0.689 | 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 3.222  | 3.646 | 3.965  |
| 18       | 0.688 | 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 3.197  | 3.610 | 3.922  |
| 19       | 0.688 | 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 3.174  | 3.579 | 3.883  |
| 20       | 0.687 | 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 3.153  | 3.552 | 3.850  |
| 21       | 0.686 | 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 3.135  | 3.527 | 3.819  |
| 22       | 0.686 | 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 3.119  | 3.505 | 3.792  |
| 23       | 0.685 | 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 3.104  | 3.485 | 3.768  |
| 24       | 0.685 | 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 3.091  | 3.467 | 3.745  |
| 25       | 0.684 | 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 3.078  | 3.450 | 3.725  |
| 26       | 0.684 | 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 3.067  | 3.435 | 3.707  |
| 27       | 0.684 | 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 3.057  | 3.421 | 3.689  |
| 28       | 0.683 | 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 3.047  | 3.408 | 3.674  |
| 29       | 0.683 | 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 3.038  | 3.396 | 3.660  |
| 30       | 0.683 | 1.310 | 1.697 | 2.042 | 2.457 | 2.750 | 3.030  | 3.385 | 3.646  |
| 40       | 0.681 | 1.303 | 1.684 | 2.021 | 2.423 | 2.704 | 2.971  | 3.307 | 3.551  |
| 60       | 0.679 | 1.296 | 1.671 | 2.000 | 2.390 | 2.660 | 2.915  | 3.232 | 3.460  |
| 120      | 0.677 | 1.289 | 1.658 | 1.980 | 2.358 | 2.617 | 2.860  | 3.160 | 3.373  |
| $\infty$ | 0.674 | 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | 2.807  | 3.090 | 3.291  |