

SYLLABUS

**Cambridge International Level 3
Pre-U Certificate in
Further Mathematics (Short Course)**

1348

For examination in 2016, 2017 and 2018

QN: 600/2568/2

Support

Cambridge provides a wide range of support for Pre-U syllabuses, which includes recommended resource lists, Teacher Guides and Example Candidate Response booklets. Teachers can access these support materials at Teacher Support <http://teachers.cie.org.uk>

Changes to syllabus for 2016, 2017 and 2018

This syllabus has been updated. The latest syllabus is version 3, published September 2016.

Significant changes to the syllabus are indicated by black vertical lines either side of the text.

Changes in version 3 of the syllabus, published September 2016

The Pre-U list of mathematical formulae and statistical tables (MF20) has been updated for examinations from June 2017. There are no substantive changes to MF20 that affect this syllabus.

Changes in version 2 of the syllabus, published September 2015

We have updated the specimen assessment materials for Paper 1. Individual questions in the specimen assessment materials have been updated in terms of wording and structure. A limited number of questions have been updated, and the overall style of assessment remains the same, but the changes better support an appropriate profile of demand. The changes do not affect the content of the qualification.

There is a correction to the equation for arc length on page 19 of the syllabus for 2016–2018. The correction has been made to the all three forms: the form for cartesian coordinates, the parametric form and the polar form.

Changes in version 1 of the syllabus, published February 2014

We reviewed and clarified the syllabus content. Additional detail in the content of the following syllabus sections has been provided:

- 'roots of polynomial equations' (page 12 of the syllabus)
- 'differential equations' (page 14 of the syllabus).

You are advised to read the whole syllabus carefully before planning your teaching programme. In addition, you are advised to refer to the updated published specimen assessment materials for examination from 2016 on our website at www.cie.org.uk

If there are any further changes to this syllabus, Cambridge will write to Centres to inform them. This syllabus is also on the Cambridge website www.cie.org.uk/cambridgepreu. The version of the syllabus on the website should always be considered as the definitive version.

Copies of Cambridge Pre-U syllabuses can be downloaded from our website www.cie.org.uk/cambridgepreu

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Introduction

Why choose Cambridge Pre-U?

Cambridge Pre-U is designed to equip learners with the skills required to make a success of their studies at university. Schools can choose from a wide range of subjects.

Cambridge Pre-U is built on a core set of educational aims to prepare learners for university admission, and also for success in higher education and beyond:

- to support independent and self-directed learning
- to encourage learners to think laterally, critically and creatively, and to acquire good problem-solving skills
- to promote comprehensive understanding of the subject through depth and rigour.

Cambridge Pre-U Short Course subjects are normally assessed at the end of a one-year programme of study in one examination series.

The Cambridge Pre-U nine-point grade set recognises the full range of learner ability.

Guided learning hours

Cambridge Pre-U syllabuses are designed on the assumption that learners have around 180 guided learning hours per Short Course subject over the duration of the course, but this is for guidance only. The number of hours may vary according to curricular practice and the learners' prior experience of the subject.

Why choose Cambridge Pre-U Further Mathematics?

- Cambridge Pre-U Further Mathematics is designed to encourage teaching and learning which enable learners to develop a positive attitude towards the subject by developing an understanding of mathematics and mathematical processes in a way that promotes confidence and enjoyment.
- The study of mathematics encourages the development of logical thought and problem-solving skills.
- Cambridge Pre-U Further Mathematics involves the acquisition of skills that can be applied in a wide range of contexts.
- Cambridge Pre-U Further Mathematics Short Course extends learners' range of mathematical skills and techniques building on the foundation of Cambridge Pre-U Mathematics as a preparation for further study.'

Prior learning

Candidates should have studied Cambridge Pre-U Mathematics. Knowledge of the pure mathematics syllabus content of Cambridge Pre-U Mathematics is assumed for Cambridge Pre-U Short Course in Further Mathematics.

Progression

Cambridge Pre-U is considered to be an excellent preparation for university, employment and life. It helps to develop the in-depth subject knowledge and understanding which are so important to universities and employers. While it is a satisfying subject in its own right, mathematics is also a prerequisite for further study in an increasing range of subjects. In conjunction with Cambridge Pre-U Mathematics, this syllabus provides a sound foundation in mathematics for higher education courses or other career pathways where mathematics is used extensively.

Syllabus aims

The aims of the syllabus, listed below, are the same for all candidates, and are to:

- enable learners to develop a range of mathematical skills and techniques
- enable learners to recognise how a situation may be represented mathematically and understand how mathematical models can be refined
- encourage learners to use mathematics as an effective means of communication, through the use of correct mathematical language and notation and through the construction of sustained logical arguments, including an appreciation of the limitations of calculator use in relation to obtaining exact solutions.

Scheme of assessment

For the Cambridge Pre-U Short Course qualification in Further Mathematics, candidates take one component.

Component	Component name	Duration	Weighting (%)	Type of assessment
Paper 1	Further Pure Mathematics	3 hours	100	Written paper, externally set and marked, 120 marks

Availability

This syllabus is examined in the June examination series.

This syllabus is available to private candidates.

Combining this with other syllabuses

Candidates can combine this syllabus in a series with any other Cambridge syllabus, except syllabuses with the same title at the same level.

Assessment objectives

A01	Manipulate mathematical expressions accurately, round answers to an appropriate degree of accuracy and understand the limitations of solutions obtained using calculators.
A02	Construct rigorous mathematical arguments and proofs through the use of precise statements and logical deduction, including extended arguments for problems presented in unstructured form.
A03	Recall, select and apply knowledge of mathematical facts, concepts and techniques in a variety of contexts.

Relationship between scheme of assessment and assessment objectives

The approximate weightings allocated to each of the assessment objectives for the whole assessment are summarised below. The table shows the assessment objectives (AO) as a percentage of the overall Cambridge Pre-U Further Mathematics Short Course qualification.

Component	AO1	AO2	AO3	Weighting of paper in overall qualification
Weighting of AO in overall qualification	36% ± 3	20% ± 3	47% ± 3	100%

Grading and reporting

The Cambridge International Level 3 Pre-U Certificates (Principal Subjects and Short Courses) are qualifications in their own right. They are acceptable as an alternative to AS Level (or other Level 3 qualifications) for entry into Higher Education or employment. Each individual Short Course Subject is graded separately on a scale of nine grades: Distinction 1, Distinction 2, Distinction 3, Merit 1, Merit 2, Merit 3, Pass 1, Pass 2 and Pass 3.

Grading Cambridge Pre-U Principal Subjects and Short Courses

Distinction	1
	2
	3
Merit	1
	2
	3
Pass	1
	2
	3

Grade descriptions

The following grade descriptions indicate the level of attainment characteristic of the middle of the given grade band. They give a general indication of the required standard at each specified grade. The descriptions should be interpreted in relation to the content outlined in the syllabus; they are not designed to define that content.

The grade awarded will depend in practice upon the extent to which the candidate has met the assessment objectives overall. Shortcomings in some aspects of the examination may be balanced by better performance in others.

Distinction (D2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with high accuracy and skill.
- If errors are made in their calculations or logic, these are mostly noticed and corrected.
- Candidates make appropriate and efficient use of calculators and other permitted resources, and are aware of any limitations to their use.
- Candidates present results to an appropriate degree of accuracy.
- Candidates use mathematical language correctly and proceed logically and rigorously through extended arguments or proofs.
- When confronted with unstructured problems, candidates can mostly devise and implement an effective solution strategy.
- Candidates recall or recognise almost all the mathematical facts, concepts and techniques that are needed, and select appropriate ones to use in a wide variety of contexts.
- Candidates are usually able to solve problems in less familiar contexts.

Merit (M2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with a reasonable level of accuracy and skill.
- Candidates often notice and correct errors in their calculations.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates usually present results to an appropriate degree of accuracy.
- Candidates use mathematical language with some skill and often proceed logically through extended arguments or proofs.
- When confronted with unstructured problems, candidates mostly devise and implement an effective solution strategy.
- Candidates recall or recognise most of the mathematical facts, concepts and techniques that are needed, and usually select appropriate ones to use in a variety of contexts.
- Candidates are often able to solve problems in less familiar contexts.

Pass (P2)

- Candidates manipulate mathematical expressions and use graphs, sketches and diagrams, all with some accuracy and skill.
- If errors are made in their calculations or logic, these are sometimes noticed and corrected.
- Candidates usually make appropriate and efficient use of calculators and other permitted resources, and are often aware of any limitations to their use.
- Candidates often present results to an appropriate degree of accuracy.
- Candidates frequently use mathematical language correctly and occasionally proceed logically through extended arguments or proofs.
- When confronted with unstructured problems candidates can sometimes make some headway with an effective solution strategy.
- Candidates recall or recognise some of the mathematical facts, concepts and techniques that are needed, and frequently select appropriate ones to use in some contexts.
- Candidates try to solve problems in less familiar contexts.

Description of components

Knowledge of the pure mathematics content of the Cambridge Pre-U Mathematics syllabus is assumed.

Paper 1 Further Pure Mathematics

Written paper, 3 hours, 120 marks

- rational functions
- roots of polynomial equations
- complex numbers
- de Moivre's theorem
- polar coordinates
- summation of series
- mathematical induction
- calculus
- hyperbolic functions
- differential equations
- vector geometry
- matrices
- groups

The paper will consist of a mixture of short, medium and longer questions with a total of 120 marks. In addition to the topics listed, candidates will be expected to apply their knowledge of pure mathematics to questions set in less familiar contexts. Candidates will be expected to answer all questions.

Use of calculators

The use of scientific calculators will be permitted. Graphic calculators will not be permitted. Candidates will be expected to be aware of the limitations inherent in the use of calculators.

Mathematical tables and formulae

Candidates will be provided with a booklet of mathematical formulae and tables for use in the examination.

Syllabus content

Paper 1 Further Pure Mathematics

Rational functions

Candidates should be able to:

- sketch graphs of simple rational functions, including the determination of oblique asymptotes, in cases where the degree of the numerator and the denominator are at most 2 (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as turning points, asymptotes and intersections with axes; the use of a discriminant in determining the set of values taken by the function is included).

Roots of polynomial equations

Candidates should be able to:

- use the relations between the symmetric functions of the roots of polynomial equations and the coefficients up to degree 4
- use a substitution to obtain an equation whose roots are related in a simple way to those of the original equation.

Complex numbers

Candidates should be able to:

- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram, for example $|z - a| < b$, $|z - a| = |z - b|$ and $\arg(z - a) = b$, and combinations of such forms
- find square roots of complex numbers in cartesian form by forming and solving appropriate quadratic equations
- recognise the modulus-argument form of a complex number, and convert a complex number in this form into cartesian form, and vice versa
- multiply and divide complex numbers in modulus-argument form, and calculate integral powers of complex numbers
- represent a complex number as $re^{i\theta}$
- use the relation $e^{i\theta} = \cos\theta + i \sin\theta$.

De Moivre's theorem

Candidates should be able to:

- understand de Moivre's theorem, for positive and negative integer exponent, in terms of the geometrical effect of multiplication and division of complex numbers
- use de Moivre's theorem to express trigonometric ratios of multiple angles in terms of powers of trigonometric ratios of the fundamental angle
- use expressions for $\sin\theta$ and $\cos\theta$ in terms of $e^{i\theta}$, e.g. for expressing powers of $\sin\theta$ and $\cos\theta$ in terms of multiple angles or for summing series
- find and use the n th roots of complex numbers.

Polar coordinates

Candidates should be able to:

- understand the relations between cartesian and polar coordinates (using the convention $r \geq 0$), and convert equations of curves from cartesian to polar form and vice versa
- sketch simple polar curves for $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$ or a subset of these intervals (detailed plotting of curves will not be expected, but sketches will generally be expected to show significant features, such as symmetry, the form of the curve at the pole and least/greatest values of r)
- use the formula $\frac{1}{2} \int_{\alpha}^{\beta} r^2 d\theta$ for the area of a sector in simple cases.

Summation of series

Candidates should be able to:

- use the standard results for $\sum r$, $\sum r^2$ and $\sum r^3$ to find related sums
- use the method of differences to obtain the sum of a finite series, e.g. by expressing the general term in partial fractions
- recognise, by direct consideration of a sum to n terms, when a series is convergent, and find the sum to infinity in such cases.

Mathematical induction

Candidates should be able to:

- use the method of mathematical induction to establish a given result (questions set may involve sums of series, divisibility or inequalities, for example)
- recognise situations where conjecture based on a limited trial followed by inductive proof is a useful strategy, and carry this out in simple cases, e.g. to find the n th derivative of xe^x .

Calculus

Candidates should be able to:

- derive and use reduction formulae for the evaluation of definite integrals in simple cases
- differentiate inverse trigonometric functions
- recognise integrals of functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to integrate associated functions by using trigonometric substitutions
- derive and use Maclaurin and Taylor series
- calculate the arc length of a curve using cartesian, parametric or polar forms
- calculate the area of a surface of revolution about either the x - or y -axis using cartesian, parametric or polar forms.

Hyperbolic functions

Candidates should be able to:

- understand the definitions of the hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$, $\operatorname{sech} x$, $\operatorname{cosech} x$ and $\operatorname{coth} x$, in terms of the exponential function
- sketch the graphs of hyperbolic functions
- use identities involving hyperbolic functions
- differentiate and integrate hyperbolic functions
- understand and use the definitions of the inverse hyperbolic functions, and derive and use their logarithmic forms
- recognise integrals of functions of the form $(x^2 + a^2)^{-\frac{1}{2}}$ and $(x^2 - a^2)^{-\frac{1}{2}}$, and integrate associated functions by using hyperbolic substitutions.

Differential equations

Candidates should be able to:

- find an integrating factor for a first-order linear differential equation, and use an integrating factor to find the general solution
- use a given substitution to reduce a first- or second-order equation to a more standard form
- recall the meaning of the terms 'complementary function' and 'particular integral' in the context of linear differential equations, and use the fact that the general solution is the sum of the complementary function and a particular integral
- find the complementary function for a first- or second-order linear differential equation with constant coefficients
- recall the form of, and find, a particular integral for a first- or second-order linear differential equation in the cases where $ax^2 + bx + c$ or ae^{bx} or $a \cos px + b \sin px$ is a suitable form, and in other cases find the appropriate coefficient(s) given a suitable form of particular integral
- use initial conditions to find a particular solution to a differential equation, and interpret the solution in the context of a problem modelled by the differential equation
- use the Taylor series method to obtain series solutions to differential equations.

Vector geometry

Candidates should be able to:

- calculate the vector product of two vectors
- understand and use the equation of a plane expressed in any of the forms $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{n} = 0$ or $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$ or $ax + by + cz = d$
- find the intersection of two planes
- calculate the shortest distance of a point from a line or from a plane
- calculate the scalar triple product, and use it to find the volume of a parallelepiped or a tetrahedron
- calculate the shortest distance between two skew lines.

Matrices

Candidates should be able to:

- carry out operations of matrix addition, subtraction and multiplication, and recognise the terms null (or zero) matrix and identity (or unit) matrix
- recall the meaning of the terms 'singular' and 'non-singular' as applied to square matrices and for 2×2 and 3×3 matrices, evaluate determinants and find inverses of non-singular matrices
- understand and use the result, for non-singular matrices, that $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
- understand the use of 2×2 matrices to represent certain geometrical transformations in the x - y plane, and in particular:
 - recognise that the matrix product \mathbf{AB} represents the transformation that results from the transformation represented by \mathbf{B} followed by the transformation represented by \mathbf{A}
 - recall how the area scale-factor of a transformation is related to the determinant of the corresponding matrix
 - find the matrix that represents a given transformation or sequence of transformations (understanding of the terms 'rotation', 'reflection', 'enlargement', 'stretch' and 'shear' will be required)
- formulate a problem involving the solution of two linear simultaneous equations in two unknowns, or three equations in three unknowns, as a problem involving the solution of a matrix equation, and vice versa
- understand the cases that may arise concerning the consistency or inconsistency of two or three linear simultaneous equations, relate them to the singularity or otherwise of the corresponding square matrix, and solve consistent systems.

Groups

Candidates should be able to:

- recall that a group consists of a set of elements together with a binary operation which is closed and associative, for which an identity exists in the set, and for which every element has an inverse in the set
- use the basic group properties to show that a given structure is, or is not, a group (questions may be set on, for example, groups of matrices, transformations, permutations, integers modulo n)
- use algebraic methods to establish properties in abstract groups in easy cases, e.g. to show that any group in which every element is self-inverse is commutative (abelian)
- recall the meaning of the term 'order' as applied both to groups and to elements of a group, and determine the order of elements in a given group
- understand the idea of a subgroup of a given group, find subgroups in simple cases, and show that given subsets are, or are not, (proper) subgroups
- recall and apply Lagrange's theorem concerning the order of a subgroup of a finite group (proof of the theorem is not required)
- recall the meaning of the term 'cyclic' as applied to groups, and show familiarity with the structure of finite groups up to order 7 (questions on groups of higher order are not excluded, but no particular prior knowledge of such groups is expected)
- understand the idea of isomorphism between groups, and determine whether given finite groups are, or are not, isomorphic.

Mathematical formulae and statistical tables

Examinations for this syllabus may use relevant formulae from the following list.

PURE MATHEMATICS

Mensuration

Surface area of sphere = $4\pi r^2$

Area of curved surface of cone = $\pi r \times$ slant height

Trigonometry

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Arithmetic Series

$$u_n = a + (n - 1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n - 1)d\}$$

Geometric Series

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_\infty = \frac{a}{1 - r} \text{ for } |r| < 1$$

Summations

$$\sum_{r=1}^n r^2 = \frac{1}{6}n(n+1)(2n+1)$$

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2$$

Binomial Series

$$\binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n, \quad (n \in \mathbb{N}), \text{ where } \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{1 \cdot 2}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{1 \cdot 2 \dots r}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Logarithms and Exponentials

$$e^{x \ln a} = a^x$$

Complex Numbers

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n(\cos n\theta + i \sin n\theta)$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

The roots of $z^n = 1$ are given by $z = e^{\frac{2\pi ki}{n}}$, for $k = 0, 1, 2, \dots, n - 1$

Maclaurin's Series

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots + \frac{x^r}{r!} f^{(r)}(0) + \dots$$

$$e^x = \exp(x) = 1 + x + \frac{x^2}{2!} + \dots + \frac{x^r}{r!} + \dots \quad \text{for all } x$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots + (-1)^{r+1} \frac{x^r}{r} + \dots \quad (-1 < x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^r \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + (-1)^r \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots + (-1)^r \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 \leq x \leq 1)$$

$$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2r+1}}{(2r+1)!} + \dots \quad \text{for all } x$$

$$\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2r}}{(2r)!} + \dots \quad \text{for all } x$$

$$\tanh^{-1} x = x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2r+1}}{2r+1} + \dots \quad (-1 < x < 1)$$

Hyperbolic Functions

$$\cosh^2 x - \sinh^2 x = 1$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh^{-1} x = \ln \{x + \sqrt{x^2 - 1}\} \quad (x \geq 1)$$

$$\sinh^{-1} x = \ln \{x + \sqrt{x^2 + 1}\}$$

$$\tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right) \quad (|x| < 1)$$

Coordinate Geometry

The perpendicular distance from (h, k) to $ax + by + c = 0$ is $\frac{|ah + bk + c|}{\sqrt{a^2 + b^2}}$

The acute angle between lines with gradients m_1 and m_2 is $\tan^{-1} \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

Trigonometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad (A \pm B \neq (k + \frac{1}{2})\pi)$$

$$\text{For } t = \tan \frac{1}{2}A : \sin A = \frac{2t}{1+t^2}, \cos A = \frac{1-t^2}{1+t^2}$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

Vectors

The resolved part of \mathbf{a} in the direction of \mathbf{b} is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|}$

The point dividing AB in the ratio $\lambda : \mu$ is $\frac{\mu\mathbf{a} + \lambda\mathbf{b}}{\lambda + \mu}$

Vector product: $\mathbf{a} \times \mathbf{b} = |\mathbf{a}||\mathbf{b}| \sin \theta \hat{\mathbf{n}} = \begin{vmatrix} \mathbf{i} & a_1 & b_1 \\ \mathbf{j} & a_2 & b_2 \\ \mathbf{k} & a_3 & b_3 \end{vmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$

If A is the point with position vector $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and the direction vector \mathbf{b} is given by

$\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$, then the straight line through A with direction vector \mathbf{b} has cartesian equation

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2} = \frac{z - a_3}{b_3} \quad (= \lambda)$$

The plane through A with normal vector $\mathbf{n} = n_1\mathbf{i} + n_2\mathbf{j} + n_3\mathbf{k}$ has cartesian equation

$$n_1x + n_2y + n_3z + d = 0 \quad \text{where } d = -\mathbf{a} \cdot \mathbf{n}$$

The plane through non-collinear points A , B and C has vector equation

$$\mathbf{r} = \mathbf{a} + \lambda(\mathbf{b} - \mathbf{a}) + \mu(\mathbf{c} - \mathbf{a}) = (1 - \lambda - \mu)\mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$$

The plane through the point with position vector \mathbf{a} and parallel to \mathbf{b} and \mathbf{c} has equation $\mathbf{r} = \mathbf{a} + s\mathbf{b} + t\mathbf{c}$

The perpendicular distance of (α, β, γ) from $n_1x + n_2y + n_3z + d = 0$ is $\frac{|n_1\alpha + n_2\beta + n_3\gamma + d|}{\sqrt{n_1^2 + n_2^2 + n_3^2}}$

Matrix Transformations

Anticlockwise rotation through θ about O : $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Reflection in the line $y = (\tan \theta)x$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

Differentiation

$\mathbf{f(x)}$	$\mathbf{f'(x)}$
$\tan kx$	$k \sec^2 kx$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\tanh^{-1} x$	$\frac{1}{1-x^2}$

Integration (+ constant; $a > 0$ where relevant)

$$f(x) \quad \int f(x) \, dx$$

$$\sec^2 kx \quad \frac{1}{k} \tan kx$$

$$\tan x \quad \ln|\sec x|$$

$$\cot x \quad \ln|\sin x|$$

$$\operatorname{cosec} x \quad -\ln|\operatorname{cosec} x + \cot x| = \ln|\tan(\frac{1}{2}x)|$$

$$\sec x \quad \ln|\sec x + \tan x| = \ln|\tan(\frac{1}{2}x + \frac{1}{4}\pi)|$$

$$\sinh x \quad \cosh x$$

$$\cosh x \quad \sinh x$$

$$\tanh x \quad \ln \cosh x$$

$$\frac{1}{\sqrt{a^2 - x^2}} \quad \sin^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$$

$$\frac{1}{a^2 + x^2} \quad \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right)$$

$$\frac{1}{\sqrt{x^2 - a^2}} \quad \cosh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 - a^2}\} \quad (x > a)$$

$$\frac{1}{\sqrt{a^2 + x^2}} \quad \sinh^{-1}\left(\frac{x}{a}\right) \text{ or } \ln\{x + \sqrt{x^2 + a^2}\}$$

$$\frac{1}{a^2 - x^2} \quad \frac{1}{2a} \ln\left|\frac{a+x}{a-x}\right| = \frac{1}{a} \tanh^{-1}\left(\frac{x}{a}\right) \quad (|x| < a)$$

$$\frac{1}{x^2 - a^2} \quad \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right|$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Area of a Sector

$$A = \frac{1}{2} \int r^2 d\theta \quad (\text{polar coordinates})$$

$$A = \frac{1}{2} \int \left(x \frac{dy}{dt} - y \frac{dx}{dt} \right) dt \quad (\text{parametric form})$$

Arc Length

$$s = \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx \quad (\text{cartesian coordinates})$$

$$s = \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt \quad (\text{parametric form})$$

$$s = \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \quad (\text{polar form})$$

Surface Area of Revolution

$$S_x = 2\pi \int y \, ds$$

$$S_y = 2\pi \int x \, ds$$

Numerical Solution of Equations

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

MECHANICS**Motion in a Circle**

Transverse velocity: $v = r\dot{\theta}$

Transverse acceleration: $\dot{v} = r\ddot{\theta}$

Radial acceleration: $-r\dot{\theta}^2 = -\frac{v^2}{r}$

PROBABILITY**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A)$$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

$$\text{Bayes' Theorem: } P(A_j|B) = \frac{P(A_j)P(B|A_j)}{\sum P(A_i)P(B|A_i)}$$

Discrete Distributions

For a discrete random variable X taking values x_i with probabilities p_i

$$\text{Expectation (mean): } E(X) = \mu = \sum x_i p_i$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \sum (x_i - \mu)^2 p_i = \sum x_i^2 p_i - \mu^2$$

$$\text{For a function } g(X) : E(g(X)) = \sum g(x_i) p_i$$

The probability generating function (P.G.F.) of X is $G_X(t) = E(t^X)$,

$$\text{and } E(X) = G'_X(1), \text{ Var}(X) = G''_X(1) + G'_X(1) - \{G'_X(1)\}^2$$

For $Z = X + Y$, where X and Y are independent: $G_Z(t) = G_X(t)G_Y(t)$

The moment generating function (M.G.F.) of X is $M_X(t) = E(e^{tX})$,

$$\text{and } E(X) = M'_X(0), E(X^n) = M_X^{(n)}(0), \text{ Var}(X) = M''_X(0) - \{M'_X(0)\}^2$$

For $Z = X + Y$, where X and Y are independent: $M_Z(t) = M_X(t)M_Y(t)$

Standard Discrete Distributions

Distribution of X	$P(X=x)$	Mean	Variance	P.G.F.	M.G.F.
Binomial $B(n, p)$	$\binom{n}{x} p^x (1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pt)^n$	$(1-p+pe^t)^n$
Poisson $Po(\lambda)$	$e^{-\lambda} \frac{\lambda^x}{x!}$	λ	λ	$e^{\lambda(t-1)}$	$e^{\lambda(e^t-1)}$
Geometric $\text{Geo}(p)$ on $1, 2, \dots$	$p(1-p)^{x-1}$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{pt}{1-(1-p)t}$	$\frac{pe^t}{1-(1-p)e^t}$

Continuous Distributions

For a continuous random variable X having probability density function f

$$\text{Expectation (mean): } E(X) = \mu = \int x f(x) dx$$

$$\text{Variance: } \text{Var}(X) = \sigma^2 = \int (x - \mu)^2 f(x) dx = \int x^2 f(x) dx - \mu^2$$

$$\text{For a function } g(X) : E(g(X)) = \int g(x) f(x) dx$$

$$\text{Cumulative distribution function: } F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$$

The moment generating function (M.G.F.) of X is $M_X(t) = E(e^{tX})$,
and $E(X) = M'_X(0)$, $E(X^n) = M_X^{(n)}(0)$, $\text{Var}(X) = M''_X(0) - \{M'_X(0)\}^2$

For $Z = X + Y$, where X and Y are independent: $M_Z(t) = M_X(t)M_Y(t)$

Standard Continuous Distributions

Distribution of X	P.D.F.	Mean	Variance	M.G.F.
Uniform (Rectangular) on $[a, b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{e^{bt} - e^{at}}{(b-a)t}$
Exponential	$\lambda e^{-\lambda x}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	$\frac{\lambda}{\lambda - t}$
Normal $N(\mu, \sigma^2)$	$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	μ	σ^2	$e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

Expectation Algebra

For independent random variables X and Y

$$E(XY) = E(X)E(Y), \quad \text{Var}(aX \pm bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$$

Sampling Distributions

For a random sample X_1, X_2, \dots, X_n of n independent observations from a distribution having mean μ and variance σ^2

$$\bar{X} \text{ is an unbiased estimator of } \mu, \text{ with } \text{Var}(\bar{X}) = \frac{\sigma^2}{n}$$

$$S^2 \text{ is an unbiased estimator of } \sigma^2, \text{ where } S^2 = \frac{\sum(X_i - \bar{X})^2}{n-1}$$

For a random sample of n observations from $N(\mu, \sigma^2)$

$$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

$$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1} \text{ (also valid in matched-pairs situations)}$$

If X is the observed number of successes in n independent Bernoulli trials in each of which the probability of success is p , and $Y = \frac{X}{n}$, then $E(Y) = p$ and $\text{Var}(Y) = \frac{p(1-p)}{n}$

For a random sample of n_x observations from $N(\mu_x, \sigma_x^2)$ and, independently, a random sample of n_y observations from $N(\mu_y, \sigma_y^2)$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0, 1)$$

If $\sigma_x^2 = \sigma_y^2 = \sigma^2$ (unknown) then $\frac{(\bar{X} - \bar{Y}) - (\mu_x - \mu_y)}{\sqrt{S_p^2 \left(\frac{1}{n_x} + \frac{1}{n_y}\right)}} \sim t_{n_x + n_y - 2}$, where $S_p^2 = \frac{(n_x - 1)S_x^2 + (n_y - 1)S_y^2}{n_x + n_y - 2}$

Correlation and Regression

For a set of n pairs of values (x_i, y_i)

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

$$S_{yy} = \Sigma(y_i - \bar{y})^2 = \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$S_{xy} = \Sigma(x_i - \bar{x})(y_i - \bar{y}) = \Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}$$

The product-moment correlation coefficient is

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\{\Sigma(x_i - \bar{x})^2\}\{\Sigma(y_i - \bar{y})^2\}}} = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\sqrt{\left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)\left(\Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}\right)}}$$

The regression coefficient of y on x is $b = \frac{S_{xy}}{S_{xx}} = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

Least squares regression line of y on x is $y = a + bx$ where $a = \bar{y} - b\bar{x}$

CUMULATIVE BINOMIAL PROBABILITIES

$n = 5$		0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	p	0.7738	0.5905	0.4437	0.4019	0.3277	0.2373	0.1681	0.1317	0.1160	0.0778	0.0503	0.0313	0.0185	0.0102	0.0053	0.0041	0.0024	0.0010	0.0003	0.0001	0.0001	0.0000	0.0000
1		0.9774	0.9185	0.8352	0.8038	0.7373	0.6328	0.5282	0.4609	0.4284	0.3370	0.2562	0.1875	0.1312	0.0870	0.0540	0.0453	0.0308	0.0156	0.0067	0.0033	0.0022	0.0005	0.0000
2		0.9988	0.9914	0.9734	0.9645	0.9421	0.8965	0.8369	0.7901	0.7648	0.6826	0.5931	0.5000	0.4069	0.3174	0.2352	0.2099	0.1631	0.1035	0.0579	0.0355	0.0266	0.0086	0.0012
3		1.0000	0.9995	0.9978	0.9967	0.9933	0.9844	0.9692	0.9547	0.9460	0.9130	0.8688	0.8125	0.7438	0.6630	0.5716	0.5391	0.4718	0.3672	0.2627	0.1962	0.1648	0.0815	0.0226
4		1.0000	1.0000	0.9999	0.9999	0.9997	0.9990	0.9976	0.9959	0.9947	0.9898	0.9815	0.9688	0.9497	0.9222	0.8840	0.8683	0.8319	0.7627	0.6723	0.5981	0.5563	0.4095	0.2262
5		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 6$		0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	p	0.7351	0.5314	0.3771	0.3349	0.2621	0.1780	0.1176	0.0878	0.0754	0.0467	0.0277	0.0156	0.0083	0.0041	0.0018	0.0014	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
1		0.9672	0.8857	0.7765	0.7368	0.6554	0.5339	0.4202	0.3512	0.3191	0.2333	0.1636	0.1094	0.0692	0.0410	0.0223	0.0178	0.0109	0.0046	0.0016	0.0007	0.0004	0.0001	0.0000
2		0.9978	0.9842	0.9527	0.9377	0.9011	0.8306	0.7443	0.6804	0.6471	0.5443	0.4415	0.3438	0.2553	0.1792	0.1174	0.1001	0.0705	0.0376	0.0170	0.0087	0.0059	0.0013	0.0001
3		0.9999	0.9987	0.9941	0.9913	0.9830	0.9624	0.9295	0.8999	0.8826	0.8208	0.7447	0.6563	0.5585	0.4557	0.3529	0.3196	0.2557	0.1694	0.0989	0.0623	0.0473	0.0159	0.0022
4		1.0000	0.9999	0.9996	0.9993	0.9984	0.9954	0.9891	0.9822	0.9777	0.9590	0.9308	0.8906	0.8364	0.7667	0.6809	0.6488	0.5798	0.4661	0.3446	0.2632	0.2235	0.1143	0.0328
5		1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9986	0.9982	0.9959	0.9917	0.9844	0.9723	0.9533	0.9246	0.9122	0.8824	0.8220	0.7379	0.6651	0.6229	0.4686	0.2649
6		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 7$		0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	p	0.6983	0.4783	0.3206	0.2791	0.2097	0.1335	0.0824	0.0585	0.0490	0.0280	0.0152	0.0078	0.0037	0.0016	0.0006	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
1		0.9556	0.8503	0.7166	0.6698	0.5767	0.4449	0.3294	0.2634	0.2338	0.1586	0.1024	0.0625	0.0357	0.0188	0.0090	0.0069	0.0038	0.0013	0.0004	0.0001	0.0001	0.0000	0.0000
2		0.9962	0.9743	0.9262	0.9042	0.8520	0.7564	0.6471	0.5706	0.5323	0.4199	0.3164	0.2266	0.1529	0.0963	0.0556	0.0453	0.0288	0.0129	0.0047	0.0020	0.0012	0.0002	0.0000
3		0.9998	0.9973	0.9879	0.9824	0.9667	0.9294	0.8740	0.8267	0.8002	0.7102	0.6083	0.5000	0.3917	0.2898	0.1998	0.1733	0.1260	0.0706	0.0333	0.0176	0.0121	0.0027	0.0002
4		1.0000	0.9998	0.9988	0.9980	0.9953	0.9871	0.9712	0.9547	0.9444	0.9037	0.8471	0.7734	0.6836	0.5801	0.4677	0.4294	0.3529	0.2436	0.1480	0.0958	0.0738	0.0257	0.0038
5		1.0000	1.0000	0.9999	0.9999	0.9996	0.9987	0.9962	0.9931	0.9910	0.9812	0.9643	0.9375	0.8976	0.8414	0.7662	0.7366	0.6706	0.5551	0.4233	0.3302	0.2834	0.1497	0.0444
6		1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9994	0.9984	0.9963	0.9922	0.9848	0.9720	0.9510	0.9415	0.9176	0.8665	0.7903	0.7209	0.6794	0.5217	0.3017
7		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 8$		0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	p	0.6634	0.4305	0.2725	0.2326	0.1678	0.1001	0.0576	0.0390	0.0319	0.0168	0.0084	0.0039	0.0017	0.0007	0.0002	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1		0.9428	0.8131	0.6572	0.6047	0.5033	0.3671	0.2553	0.1951	0.1691	0.1064	0.0632	0.0352	0.0181	0.0085	0.0036	0.0026	0.0013	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
2		0.9942	0.9619	0.8948	0.8652	0.7969	0.6785	0.5518	0.4682	0.4278	0.3154	0.2201	0.1445	0.0885	0.0498	0.0253	0.0197	0.0113	0.0042	0.0012	0.0004	0.0002	0.0000	0.0000
3		0.9996	0.9950	0.9786	0.9693	0.9437	0.8862	0.8059	0.7414	0.7064	0.5941	0.4770	0.3633	0.2604	0.1737	0.1061	0.0879	0.0580	0.0273	0.0104	0.0046	0.0029	0.0004	0.0000
4		1.0000	0.9996	0.9971	0.9954	0.9896	0.9727	0.9420	0.9121	0.8939	0.8263	0.7396	0.6367	0.5230	0.4059	0.2936	0.2586	0.1941	0.1138	0.0563	0.0307	0.0214	0.0050	0.0004
5		1.0000	1.0000	0.9998	0.9996	0.9988	0.9958	0.9887	0.9803	0.9747	0.9502	0.9115	0.8555	0.7799	0.6846	0.5722	0.5318	0.4482	0.3215	0.2031	0.1348	0.1052	0.0381	0.0058
6		1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9974	0.9964	0.9915	0.9819	0.9648	0.9368	0.8936	0.8309	0.8049	0.7447	0.6329	0.4967	0.3953	0.3428	0.1869	0.0572
7		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9998	0.9993	0.9983	0.9961	0.9916	0.9832	0.9681	0.9610	0.9424	0.8999	0.8322	0.7674	0.7275	0.5695	0.3366
8		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 9$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95	
$x = 0$	0.6302	0.3874	0.2316	0.1938	0.1342	0.0751	0.0404	0.0260	0.0207	0.0101	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9288	0.7748	0.5995	0.5427	0.4362	0.3003	0.1960	0.1431	0.1211	0.0705	0.0385	0.0195	0.0091	0.0038	0.0014	0.0010	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9916	0.9470	0.8591	0.8217	0.7382	0.6007	0.4628	0.3772	0.3373	0.2318	0.1495	0.0898	0.0498	0.0250	0.0112	0.0083	0.0043	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.9994	0.9917	0.9661	0.9520	0.9144	0.8343	0.7297	0.6503	0.6089	0.4826	0.3614	0.2539	0.1658	0.0994	0.0536	0.0424	0.0253	0.0100	0.0031	0.0011	0.0006	0.0001	0.0000	0.0000
4	1.0000	0.9991	0.9944	0.9910	0.9804	0.9511	0.9012	0.8552	0.8283	0.7334	0.6214	0.5000	0.3786	0.2666	0.1717	0.1448	0.0988	0.0489	0.0196	0.0090	0.0056	0.0009	0.0000	0.0000
5	1.0000	0.9999	0.9994	0.9989	0.9969	0.9900	0.9747	0.9576	0.9464	0.9006	0.8342	0.7461	0.6386	0.5174	0.3911	0.3497	0.2703	0.1657	0.0856	0.0480	0.0339	0.0083	0.0006	0.0000
6	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9957	0.9917	0.9888	0.9750	0.9502	0.9102	0.8505	0.7682	0.6627	0.6228	0.5372	0.3993	0.2618	0.1783	0.1409	0.0530	0.0084	0.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9986	0.9962	0.9909	0.9805	0.9615	0.9295	0.8789	0.8569	0.8040	0.6997	0.5638	0.4573	0.4005	0.2252	0.0712	0.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9992	0.9980	0.9954	0.9899	0.9793	0.9740	0.9596	0.9249	0.8658	0.8062	0.7684	0.6126	0.3698	0.0000	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 10$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95	
$x = 0$	0.5987	0.3487	0.1969	0.1615	0.1074	0.0563	0.0282	0.0173	0.0135	0.0060	0.0025	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.9139	0.7361	0.5443	0.4845	0.3758	0.2440	0.1493	0.1040	0.0860	0.0464	0.0233	0.0107	0.0045	0.0017	0.0005	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9885	0.9298	0.8202	0.7752	0.6778	0.5256	0.3828	0.2991	0.2616	0.1673	0.0996	0.0547	0.0274	0.0123	0.0048	0.0034	0.0016	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9990	0.9872	0.9500	0.9303	0.8791	0.7759	0.6496	0.5593	0.5138	0.3823	0.2660	0.1719	0.1020	0.0548	0.0260	0.0197	0.0106	0.0035	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
4	0.9999	0.9984	0.9901	0.9845	0.9672	0.9219	0.8497	0.7869	0.7515	0.6331	0.5044	0.3770	0.2616	0.1662	0.0949	0.0766	0.0473	0.0197	0.0064	0.0024	0.0014	0.0001	0.0000	0.0000
5	1.0000	0.9999	0.9986	0.9976	0.9936	0.9803	0.9527	0.9234	0.9051	0.8338	0.7384	0.6230	0.4956	0.3669	0.2485	0.2131	0.1503	0.0781	0.0328	0.0155	0.0099	0.0016	0.0001	0.0000
6	1.0000	1.0000	0.9999	0.9997	0.9991	0.9965	0.9894	0.9803	0.9740	0.9452	0.8980	0.8281	0.7340	0.6177	0.4862	0.4407	0.3504	0.2241	0.1209	0.0697	0.0500	0.0128	0.0010	0.0000
7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9966	0.9952	0.9877	0.9726	0.9453	0.9004	0.8327	0.7384	0.7009	0.6172	0.4744	0.3222	0.2248	0.1798	0.0702	0.0115	0.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9995	0.9983	0.9955	0.9893	0.9767	0.9536	0.9140	0.8960	0.8507	0.7560	0.6242	0.5155	0.4557	0.2639	0.0861	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9975	0.9940	0.9865	0.9827	0.9718	0.9437	0.8926	0.8385	0.8031	0.6513	0.4013	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 12$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95	
$x = 0$	0.5404	0.2824	0.1422	0.1122	0.0687	0.0317	0.0138	0.0077	0.0057	0.0022	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8816	0.6590	0.4435	0.3813	0.2749	0.1584	0.0850	0.0540	0.0424	0.0196	0.0083	0.0032	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9804	0.8891	0.7358	0.6774	0.5583	0.3907	0.2528	0.1811	0.1513	0.0834	0.0421	0.0193	0.0079	0.0028	0.0008	0.0005	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9978	0.9744	0.9078	0.8748	0.7946	0.6488	0.4925	0.3931	0.3467	0.2253	0.1345	0.0730	0.0356	0.0153	0.0056	0.0039	0.0017	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9998	0.9957	0.9761	0.9636	0.9274	0.8424	0.7237	0.6315	0.5833	0.4382	0.3044	0.1938	0.1117	0.0573	0.0255	0.0188	0.0095	0.0028	0.0006	0.0002	0.0001	0.0000	0.0000	0.0000
5	1.0000	0.9995	0.9954	0.9921	0.9806	0.9456	0.8822	0.8223	0.7873	0.6652	0.5269	0.3872	0.2607	0.1582	0.0846	0.0664	0.0386	0.0143	0.0039	0.0013	0.0007	0.0001	0.0000	0.0000
6	1.0000	0.9999	0.9993	0.9987	0.9961	0.9857	0.9614	0.9336	0.9154	0.8418	0.7393	0.6128	0.4731	0.3348	0.2127	0.1777	0.1178	0.0544	0.0194	0.0079	0.0046	0.0005	0.0000	0.0000
7	1.0000	1.0000	0.9999	0.9998	0.9994	0.9972	0.9905	0.9812	0.9745	0.9427	0.8883	0.8062	0.6956	0.5618	0.4167	0.3685	0.2763	0.1576	0.0726	0.0364	0.0239	0.0043	0.0002	0.0000
8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9961	0.9944	0.9847	0.9644	0.9270	0.8655	0.7747	0.6533	0.6069	0.5075	0.3512	0.2054	0.1252	0.0922	0.0256	0.0022	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9995	0.9992	0.9972	0.9921	0.9807	0.9579	0.9166	0.8487	0.8189	0.7472	0.6093	0.4417	0.3226	0.2642	0.1109	0.0196	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9989	0.9968	0.9917	0.9804	0.9576	0.9460	0.9150	0.8416	0.7251	0.6187	0.5565	0.3410	0.1184	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9978	0.9943	0.9923	0.9862	0.9683	0.9313	0.8878	0.8578	0.7176	0.4596	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 14$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4877	0.2288	0.1028	0.0779	0.0440	0.0178	0.0068	0.0034	0.0024	0.0008	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8470	0.5846	0.3567	0.2960	0.1979	0.1010	0.0475	0.0274	0.0205	0.0081	0.0029	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9699	0.8416	0.6479	0.5795	0.4481	0.2811	0.1608	0.1053	0.0839	0.0398	0.0170	0.0065	0.0022	0.0006	0.0001	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9958	0.9559	0.8535	0.8063	0.6982	0.5213	0.3552	0.2612	0.2205	0.1243	0.0632	0.0287	0.0114	0.0039	0.0011	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9996	0.9908	0.9533	0.9310	0.8702	0.7415	0.5842	0.4755	0.4227	0.2793	0.1672	0.0898	0.0426	0.0175	0.0060	0.0040	0.0017	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
5	1.0000	0.9985	0.9885	0.9809	0.9561	0.8883	0.7805	0.6898	0.6405	0.4859	0.3373	0.2120	0.1189	0.0583	0.0243	0.0174	0.0083	0.0022	0.0004	0.0001	0.0000	0.0000	0.0000
6	1.0000	0.9998	0.9978	0.9959	0.9884	0.9617	0.9067	0.8505	0.8164	0.6925	0.5461	0.3953	0.2586	0.1501	0.0753	0.0576	0.0315	0.0103	0.0024	0.0007	0.0003	0.0000	0.0000
7	1.0000	1.0000	0.9997	0.9993	0.9976	0.9897	0.9685	0.9424	0.9247	0.8499	0.7414	0.6047	0.4539	0.3075	0.1836	0.1495	0.0933	0.0383	0.0116	0.0041	0.0022	0.0002	0.0000
8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9978	0.9917	0.9826	0.9757	0.9417	0.8811	0.7880	0.6627	0.5141	0.3595	0.3102	0.2195	0.1117	0.0439	0.0191	0.0115	0.0015	0.0000
9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9960	0.9940	0.9825	0.9574	0.9102	0.8328	0.7207	0.5773	0.5245	0.4158	0.2585	0.1298	0.0690	0.0467	0.0092	0.0004
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9989	0.9961	0.9886	0.9713	0.9368	0.8757	0.7795	0.7388	0.6448	0.4787	0.3018	0.1937	0.1465	0.0441	0.0042
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9978	0.9935	0.9830	0.9602	0.9161	0.8947	0.8392	0.7189	0.5519	0.4205	0.3521	0.1584	0.0301
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9971	0.9919	0.9795	0.9726	0.9525	0.8990	0.8021	0.7040	0.6433	0.4154	0.1530
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9992	0.9976	0.9966	0.9932	0.9822	0.9560	0.9221	0.8972	0.7712	0.5123
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$n = 16$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.4401	0.1853	0.0743	0.0541	0.0281	0.0100	0.0033	0.0015	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.8108	0.5147	0.2839	0.2272	0.1407	0.0635	0.0261	0.0137	0.0098	0.0033	0.0010	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9571	0.7892	0.5614	0.4868	0.3518	0.1971	0.0994	0.0594	0.0451	0.0183	0.0066	0.0021	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9930	0.9316	0.7899	0.7291	0.5981	0.4050	0.2459	0.1659	0.1339	0.0651	0.0281	0.0106	0.0035	0.0009	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9991	0.9830	0.9209	0.8866	0.7982	0.6302	0.4499	0.3391	0.2892	0.1666	0.0853	0.0384	0.0149	0.0049	0.0013	0.0008	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9999	0.9967	0.9765	0.9622	0.9183	0.8103	0.6598	0.5469	0.4900	0.3288	0.1976	0.1051	0.0486	0.0191	0.0062	0.0040	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9995	0.9944	0.9899	0.9733	0.9204	0.8247	0.7374	0.6881	0.5272	0.3660	0.2272	0.1241	0.0583	0.0229	0.0159	0.0071	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9999	0.9989	0.9979	0.9930	0.9729	0.9256	0.8735	0.8406	0.7161	0.5629	0.4018	0.2559	0.1423	0.0671	0.0500	0.0257	0.0075	0.0015	0.0004	0.0002	0.0000	0.0000
8	1.0000	1.0000	0.9998	0.9996	0.9985	0.9925	0.9743	0.9500	0.9329	0.8577	0.7441	0.5982	0.4371	0.2839	0.1594	0.1265	0.0744	0.0271	0.0070	0.0021	0.0011	0.0001	0.0000
9	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9841	0.9771	0.9417	0.8759	0.7728	0.6340	0.4728	0.3119	0.2626	0.1753	0.0796	0.0267	0.0101	0.0056	0.0005	0.0000
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9960	0.9938	0.9809	0.9514	0.8949	0.8024	0.6712	0.5100	0.4531	0.3402	0.1897	0.0817	0.0378	0.0235	0.0033	0.0001
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9992	0.9987	0.9951	0.9851	0.9616	0.9147	0.8334	0.7108	0.6609	0.5501	0.3698	0.2018	0.1134	0.0791	0.0170	0.0009
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9991	0.9965	0.9894	0.9719	0.9349	0.8661	0.8341	0.7541	0.5950	0.4019	0.2709	0.2101	0.0684	0.0070	
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9994	0.9979	0.9934	0.9817	0.9549	0.9406	0.9006	0.8029	0.6482	0.5132	0.4386	0.2108	0.0429
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9967	0.9902	0.9863	0.9739	0.9365	0.8593	0.7728	0.7161	0.4853	0.1892	
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9985	0.9967	0.9900	0.9719	0.9459	0.9257	0.8147	0.5599	
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 18$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3972	0.1501	0.0536	0.0376	0.0180	0.0056	0.0016	0.0007	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7735	0.4503	0.2241	0.1728	0.0991	0.0395	0.0142	0.0068	0.0046	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9419	0.7338	0.4797	0.4027	0.2713	0.1353	0.0600	0.0326	0.0236	0.0082	0.0025	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9891	0.9018	0.7202	0.6479	0.5010	0.3057	0.1646	0.1017	0.0783	0.0328	0.0120	0.0038	0.0010	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9985	0.9718	0.8794	0.8318	0.7164	0.5187	0.3327	0.2311	0.1886	0.0942	0.0411	0.0154	0.0049	0.0013	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9998	0.9936	0.9581	0.9347	0.8671	0.7175	0.5344	0.4122	0.3550	0.2088	0.1077	0.0481	0.0183	0.0058	0.0014	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9988	0.9882	0.9794	0.9487	0.8610	0.7217	0.6085	0.5491	0.3743	0.2258	0.1189	0.0537	0.0203	0.0062	0.0039	0.0014	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9998	0.9973	0.9947	0.9837	0.9431	0.8593	0.7767	0.7283	0.5634	0.3915	0.2403	0.1280	0.0576	0.0212	0.0144	0.0061	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000
8	1.0000	1.0000	0.9995	0.9989	0.9957	0.9807	0.9404	0.8924	0.8609	0.7368	0.5778	0.4073	0.2527	0.1347	0.0597	0.0433	0.0210	0.0054	0.0009	0.0002	0.0001	0.0000	0.0000
9	1.0000	1.0000	0.9999	0.9998	0.9991	0.9946	0.9790	0.9567	0.9403	0.8653	0.7473	0.5927	0.4222	0.2632	0.1391	0.1076	0.0596	0.0193	0.0043	0.0011	0.0005	0.0000	0.0000
10	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9856	0.9788	0.9424	0.8720	0.7597	0.6085	0.4366	0.2717	0.2233	0.1407	0.0569	0.0163	0.0053	0.0027	0.0002	0.0000
11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9961	0.9938	0.9797	0.9463	0.8811	0.7742	0.6257	0.4509	0.3915	0.2783	0.1390	0.0513	0.0206	0.0118	0.0012	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9986	0.9942	0.9817	0.9519	0.8923	0.7912	0.6450	0.5878	0.4656	0.2825	0.1329	0.0653	0.0419	0.0064	0.0002
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9951	0.9846	0.9589	0.9058	0.8114	0.7689	0.6673	0.4813	0.2836	0.1682	0.1206	0.0282	0.0015
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9990	0.9962	0.9880	0.9672	0.9217	0.8983	0.8354	0.6943	0.4990	0.3521	0.2798	0.0982	0.0109
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9975	0.9918	0.9764	0.9674	0.9400	0.8647	0.7287	0.5973	0.5203	0.2662	0.0581
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9987	0.9954	0.9932	0.9858	0.9605	0.9009	0.8272	0.7759	0.5497	0.2265
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9993	0.9984	0.9944	0.9820	0.9624	0.9464	0.8499
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
$n = 20$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.3585	0.1216	0.0388	0.0261	0.0115	0.0032	0.0008	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.7358	0.3917	0.1756	0.1304	0.0692	0.0243	0.0076	0.0033	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.9245	0.6769	0.4049	0.3287	0.2061	0.0913	0.0355	0.0176	0.0121	0.0036	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9841	0.8670	0.6477	0.5665	0.4114	0.2252	0.1071	0.0604	0.0444	0.0160	0.0049	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9974	0.9568	0.8298	0.7687	0.6296	0.4148	0.2375	0.1515	0.1182	0.0510	0.0189	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9997	0.9887	0.9327	0.8982	0.8042	0.6172	0.4164	0.2972	0.2454	0.1256	0.0553	0.0207	0.0064	0.0016	0.0003	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	1.0000	0.9976	0.9781	0.9629	0.9133	0.7858	0.6080	0.4793	0.4166	0.2500	0.1299	0.0577	0.0214	0.0065	0.0015	0.0009	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9996	0.9941	0.9887	0.9679	0.8982	0.7723	0.6615	0.6010	0.4159	0.2520	0.1316	0.0580	0.0210	0.0060	0.0037	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9999	0.9987	0.9972	0.9900	0.9591	0.8867	0.8095	0.7624	0.5956	0.4143	0.2517	0.1308	0.0565	0.0196	0.0130	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
9	1.0000	1.0000	0.9998	0.9994	0.9974	0.9861	0.9520	0.9081	0.8782	0.7553	0.5914	0.4119	0.2493	0.1275	0.0532	0.0376	0.0171	0.0039	0.0006	0.0001	0.0000	0.0000	0.0000
10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9961	0.9829	0.9624	0.9468	0.8725	0.7507	0.5881	0.4086	0.2447	0.1218	0.0919	0.0480	0.0139	0.0026	0.0006	0.0002	0.0000	0.0000
11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9870	0.9804	0.9435	0.8692	0.7483	0.5857	0.4044	0.2376	0.1905	0.1133	0.0409	0.0100	0.0028	0.0013	0.0001	0.0000
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9963	0.9940	0.9790	0.9420	0.8684	0.7480	0.5841	0.3990	0.3385	0.2277	0.1018	0.0321	0.0113	0.0059	0.0004	0.0000
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9991	0.9985	0.9935	0.9786	0.9423	0.8701	0.7500	0.5834	0.5207	0.3920	0.2142	0.0867	0.0371	0.0219	0.0024	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9984	0.9936	0.9793	0.9447	0.8744	0.7546	0.7028	0.5836	0.3828	0.1958	0.1018	0.0673	0.0113	0.0003
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9985	0.9941	0.9811	0.9490	0.8818	0.8485	0.7625	0.5852	0.3704	0.2313	0.1702	0.0432	0.0026
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9987	0.9951	0.9840	0.9556	0.9396	0.8929	0.7748	0.5886	0.4335	0.3523	0.1330	0.0159
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9991	0.9964	0.9879	0.9824	0.9645	0.9087	0.7939	0.6713	0.5951	0.3231	0.0755
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9979	0.9967	0.9924	0.9757	0.9308	0.8696	0.8244	0.6083	0.2642
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9997	0.9992	0.9968	0.9885	0.9739	0.9612	0.8784	0.6415
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 25$																							
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95
$x = 0$	0.2774	0.0718	0.0172	0.0105	0.0038	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.6424	0.2712	0.0931	0.0629	0.0274	0.0070	0.0016	0.0005	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.8729	0.5371	0.2537	0.1887	0.0982	0.0321	0.0090	0.0035	0.0021	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.9659	0.7636	0.4711	0.3816	0.2340	0.0962	0.0332	0.0149	0.0097	0.0024	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4	0.9928	0.9020	0.6821	0.5937	0.4207	0.2137	0.0905	0.0462	0.0320	0.0095	0.0023	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
5	0.9988	0.9666	0.8385	0.7720	0.6167	0.3783	0.1935	0.1120	0.0826	0.0294	0.0086	0.0020	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
6	0.9998	0.9905	0.9305	0.8908	0.7800	0.5611	0.3407	0.2215	0.1734	0.0736	0.0258	0.0073	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
7	1.0000	0.9977	0.9745	0.9553	0.8909	0.7265	0.5118	0.3703	0.3061	0.1536	0.0639	0.0216	0.0058	0.0012	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8	1.0000	0.9995	0.9920	0.9843	0.9532	0.8506	0.6769	0.5376	0.4668	0.2735	0.1340	0.0539	0.0174	0.0043	0.0008	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9	1.0000	0.9999	0.9979	0.9953	0.9827	0.9287	0.8106	0.6956	0.6303	0.4246	0.2424	0.1148	0.0440	0.0132	0.0029	0.0016	0.0005	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	1.0000	1.0000	0.9995	0.9988	0.9944	0.9703	0.9022	0.8220	0.7712	0.5858	0.3843	0.2122	0.0960	0.0344	0.0093	0.0056	0.0018	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
11	1.0000	1.0000	0.9999	0.9997	0.9985	0.9893	0.9558	0.9082	0.8746	0.7323	0.5426	0.3450	0.1827	0.0778	0.0255	0.0164	0.0060	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000
12	1.0000	1.0000	1.0000	0.9999	0.9996	0.9966	0.9825	0.9585	0.9396	0.8462	0.6937	0.5000	0.3063	0.1538	0.0604	0.0415	0.0175	0.0034	0.0004	0.0001	0.0000	0.0000	0.0000
13	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9940	0.9836	0.9745	0.9222	0.8173	0.6550	0.4574	0.2677	0.1254	0.0918	0.0442	0.0107	0.0015	0.0003	0.0001	0.0000	0.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9982	0.9944	0.9907	0.9656	0.9040	0.7878	0.6157	0.4142	0.2288	0.1780	0.0978	0.0297	0.0056	0.0012	0.0005	0.0000	0.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9984	0.9971	0.9868	0.9560	0.8852	0.7576	0.5754	0.3697	0.3044	0.1894	0.0713	0.0173	0.0047	0.0021	0.0001	0.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9992	0.9957	0.9826	0.9461	0.8660	0.7265	0.5332	0.4624	0.3231	0.1494	0.0468	0.0157	0.0080	0.0005	0.0000
17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9988	0.9942	0.9784	0.9361	0.8464	0.6939	0.6297	0.4882	0.2735	0.1091	0.0447	0.0255	0.0023	0.0000
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9927	0.9742	0.9264	0.8266	0.7785	0.6593	0.4389	0.2200	0.1092	0.0695	0.0095	0.0002
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9980	0.9914	0.9706	0.9174	0.8880	0.8065	0.6217	0.3833	0.2280	0.1615	0.0334	0.0012
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9977	0.9905	0.9680	0.9538	0.9095	0.7863	0.5793	0.4063	0.3179	0.0980	0.0072
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9976	0.9903	0.9851	0.9668	0.9038	0.7660	0.6184	0.5289	0.2364	0.0341
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9979	0.9965	0.9910	0.9679	0.9018	0.8113	0.7463	0.4629	0.1271
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9984	0.9930	0.9726	0.9371	0.9069	0.7288	0.3576
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9962	0.9895	0.9828	0.9282	0.7226
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE BINOMIAL PROBABILITIES

$n = 30$																								
p	0.05	0.1	0.15	1/6	0.2	0.25	0.3	1/3	0.35	0.4	0.45	0.5	0.55	0.6	0.65	2/3	0.7	0.75	0.8	5/6	0.85	0.9	0.95	
$x = 0$	0.2146	0.0424	0.0076	0.0042	0.0012	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
1	0.5535	0.1837	0.0480	0.0295	0.0105	0.0020	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
2	0.8122	0.4114	0.1514	0.1028	0.0442	0.0106	0.0021	0.0007	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
3	0.9392	0.6474	0.3217	0.2396	0.1227	0.0374	0.0093	0.0033	0.0019	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
4	0.9844	0.8245	0.5245	0.4243	0.2552	0.0979	0.0302	0.0122	0.0075	0.0015	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
5	0.9967	0.9268	0.7106	0.6164	0.4275	0.2026	0.0766	0.0355	0.0233	0.0057	0.0011	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
6	0.9994	0.9742	0.8474	0.7765	0.6070	0.3481	0.1595	0.0838	0.0586	0.0172	0.0040	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
7	0.9999	0.9922	0.9302	0.8863	0.7608	0.5143	0.2814	0.1668	0.1238	0.0435	0.0121	0.0026	0.0004	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
8	1.0000	0.9980	0.9722	0.9494	0.8713	0.6736	0.4315	0.2860	0.2247	0.0940	0.0312	0.0081	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
9	1.0000	0.9995	0.9903	0.9803	0.9389	0.8034	0.5888	0.4317	0.3575	0.1763	0.0694	0.0214	0.0050	0.0009	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
10	1.0000	0.9999	0.9971	0.9933	0.9744	0.8943	0.7304	0.5848	0.5078	0.2915	0.1350	0.0494	0.0138	0.0029	0.0004	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
11	1.0000	1.0000	0.9992	0.9980	0.9905	0.9493	0.8407	0.7239	0.6548	0.4311	0.2327	0.1002	0.0334	0.0083	0.0014	0.0007	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	
12	1.0000	1.0000	0.9998	0.9995	0.9969	0.9784	0.9155	0.8340	0.7802	0.5785	0.3592	0.1808	0.0714	0.0212	0.0045	0.0025	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	
13	1.0000	1.0000	1.0000	0.9999	0.9991	0.9918	0.9599	0.9102	0.8737	0.7145	0.5025	0.2923	0.1356	0.0481	0.0124	0.0072	0.0021	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	
14	1.0000	1.0000	1.0000	1.0000	0.9998	0.9973	0.9831	0.9565	0.9348	0.8246	0.6448	0.4278	0.2309	0.0971	0.0301	0.0188	0.0064	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	
15	1.0000	1.0000	1.0000	1.0000	0.9999	0.9992	0.9936	0.9812	0.9699	0.9029	0.7691	0.5722	0.3552	0.1754	0.0652	0.0435	0.0169	0.0027	0.0002	0.0000	0.0000	0.0000	0.0000	
16	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9979	0.9928	0.9876	0.9519	0.8644	0.7077	0.4975	0.2855	0.1263	0.0898	0.0401	0.0082	0.0009	0.0001	0.0000	0.0000	0.0000	
17	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9975	0.9955	0.9788	0.9286	0.8192	0.6408	0.4215	0.2198	0.1660	0.0845	0.0216	0.0031	0.0005	0.0002	0.0000	0.0000	
18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9993	0.9986	0.9917	0.9666	0.8998	0.7673	0.5689	0.3452	0.2761	0.1593	0.0507	0.0095	0.0020	0.0008	0.0000	0.0000	
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9996	0.9971	0.9862	0.9506	0.8650	0.7085	0.4922	0.4152	0.2696	0.1057	0.0256	0.0067	0.0029	0.0001	0.0000	
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991	0.9950	0.9786	0.9306	0.8237	0.6425	0.5683	0.4112	0.1966	0.0611	0.0197	0.0097	0.0005	0.0000	
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9984	0.9919	0.9688	0.9060	0.7753	0.7140	0.5685	0.3264	0.1287	0.0506	0.0278	0.0020	0.0000	
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996	0.9974	0.9879	0.9565	0.8762	0.8332	0.7186	0.4857	0.2392	0.1137	0.0698	0.0078	0.0001	
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9960	0.9828	0.9414	0.9162	0.8405	0.6519	0.3930	0.2235	0.1526	0.0258	0.0006	
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9989	0.9943	0.9767	0.9645	0.9234	0.7974	0.5725	0.3836	0.2894	0.0732	0.0033	
25	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9985	0.9925	0.9878	0.9698	0.9021	0.7448	0.5757	0.4755	0.1755	0.0156	
26	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9981	0.9967	0.9907	0.9626	0.8773	0.7604	0.6783	0.3526	0.0608	
27	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9993	0.9979	0.9894	0.9558	0.8972	0.8486	0.5886	0.1878	
28	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9980	0.9895	0.9705	0.9520	0.8163	0.4465
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9988	0.9958	0.9924	0.9576	0.7854	
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	

CUMULATIVE POISSON PROBABILITIES

λ	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
$x = 0$	0.9900	0.9802	0.9704	0.9608	0.9512	0.9418	0.9324	0.9231	0.9139
1	1.0000	0.9998	0.9996	0.9992	0.9988	0.9983	0.9977	0.9970	0.9962
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$x = 0$	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493	0.4066
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088	0.7725
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526	0.9371
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909	0.9865
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986	0.9977
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9997
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90
$x = 0$	0.3679	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496
1	0.7358	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337
2	0.9197	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037
3	0.9810	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747
4	0.9963	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559
5	0.9994	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868
6	0.9999	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966
7	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	2.00	2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90
$x = 0$	0.1353	0.1225	0.1108	0.1003	0.0907	0.0821	0.0743	0.0672	0.0608	0.0550
1	0.4060	0.3796	0.3546	0.3309	0.3084	0.2873	0.2674	0.2487	0.2311	0.2146
2	0.6767	0.6496	0.6227	0.5960	0.5697	0.5438	0.5184	0.4936	0.4695	0.4460
3	0.8571	0.8386	0.8194	0.7993	0.7787	0.7576	0.7360	0.7141	0.6919	0.6696
4	0.9473	0.9379	0.9275	0.9162	0.9041	0.8912	0.8774	0.8629	0.8477	0.8318
5	0.9834	0.9796	0.9751	0.9700	0.9643	0.9580	0.9510	0.9433	0.9349	0.9258
6	0.9955	0.9941	0.9925	0.9906	0.9884	0.9858	0.9828	0.9794	0.9756	0.9713
7	0.9989	0.9985	0.9980	0.9974	0.9967	0.9958	0.9947	0.9934	0.9919	0.9901
8	0.9998	0.9997	0.9995	0.9994	0.9991	0.9989	0.9985	0.9981	0.9976	0.9969
9	1.0000	0.9999	0.9999	0.9999	0.9998	0.9997	0.9996	0.9995	0.9993	0.9991
10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9998	0.9998
11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	3.00	3.10	3.20	3.30	3.40	3.50	3.60	3.70	3.80	3.90
$x = 0$	0.0498	0.0450	0.0408	0.0369	0.0334	0.0302	0.0273	0.0247	0.0224	0.0202
1	0.1991	0.1847	0.1712	0.1586	0.1468	0.1359	0.1257	0.1162	0.1074	0.0992
2	0.4232	0.4012	0.3799	0.3594	0.3397	0.3208	0.3027	0.2854	0.2689	0.2531
3	0.6472	0.6248	0.6025	0.5803	0.5584	0.5366	0.5152	0.4942	0.4735	0.4532
4	0.8153	0.7982	0.7806	0.7626	0.7442	0.7254	0.7064	0.6872	0.6678	0.6484
5	0.9161	0.9057	0.8946	0.8829	0.8705	0.8576	0.8441	0.8301	0.8156	0.8006
6	0.9665	0.9612	0.9554	0.9490	0.9421	0.9347	0.9267	0.9182	0.9091	0.8995
7	0.9881	0.9858	0.9832	0.9802	0.9769	0.9733	0.9692	0.9648	0.9599	0.9546
8	0.9962	0.9953	0.9943	0.9931	0.9917	0.9901	0.9883	0.9863	0.9840	0.9815
9	0.9989	0.9986	0.9982	0.9978	0.9973	0.9967	0.9960	0.9952	0.9942	0.9931
10	0.9997	0.9996	0.9995	0.9994	0.9992	0.9990	0.9987	0.9984	0.9981	0.9977
11	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9996	0.9995	0.9994	0.9993
12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9998	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

λ	4.00	4.10	4.20	4.30	4.40	4.50	4.60	4.70	4.80	4.90
$x = 0$	0.0183	0.0166	0.0150	0.0136	0.0123	0.0111	0.0101	0.0091	0.0082	0.0074
1	0.0916	0.0845	0.0780	0.0719	0.0663	0.0611	0.0563	0.0518	0.0477	0.0439
2	0.2381	0.2238	0.2102	0.1974	0.1851	0.1736	0.1626	0.1523	0.1425	0.1333
3	0.4335	0.4142	0.3954	0.3772	0.3594	0.3423	0.3257	0.3097	0.2942	0.2793
4	0.6288	0.6093	0.5898	0.5704	0.5512	0.5321	0.5132	0.4946	0.4763	0.4582
5	0.7851	0.7693	0.7531	0.7367	0.7199	0.7029	0.6858	0.6684	0.6510	0.6335
6	0.8893	0.8786	0.8675	0.8558	0.8436	0.8311	0.8180	0.8046	0.7908	0.7767
7	0.9489	0.9427	0.9361	0.9290	0.9214	0.9134	0.9049	0.8960	0.8867	0.8769
8	0.9786	0.9755	0.9721	0.9683	0.9642	0.9597	0.9549	0.9497	0.9442	0.9382
9	0.9919	0.9905	0.9889	0.9871	0.9851	0.9829	0.9805	0.9778	0.9749	0.9717
10	0.9972	0.9966	0.9959	0.9952	0.9943	0.9933	0.9922	0.9910	0.9896	0.9880
11	0.9991	0.9989	0.9986	0.9983	0.9980	0.9976	0.9971	0.9966	0.9960	0.9953
12	0.9997	0.9997	0.9996	0.9995	0.9993	0.9992	0.9990	0.9988	0.9986	0.9983
13	0.9999	0.9999	0.9999	0.9998	0.9998	0.9997	0.9997	0.9996	0.9995	0.9994
14	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0.9999	0.9998
15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

λ	5.00	5.50	6.00	6.50	7.00	7.50	8.00	8.50	9.00	9.50
$x = 0$	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006	0.0003	0.0002	0.0001	0.0001
1	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047	0.0030	0.0019	0.0012	0.0008
2	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203	0.0138	0.0093	0.0062	0.0042
3	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591	0.0424	0.0301	0.0212	0.0149
4	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321	0.0996	0.0744	0.0550	0.0403
5	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414	0.1912	0.1496	0.1157	0.0885
6	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782	0.3134	0.2562	0.2068	0.1649
7	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246	0.4530	0.3856	0.3239	0.2687
8	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620	0.5925	0.5231	0.4557	0.3918
9	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764	0.7166	0.6530	0.5874	0.5218
10	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622	0.8159	0.7634	0.7060	0.6453
11	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208	0.8881	0.8487	0.8030	0.7520
12	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573	0.9362	0.9091	0.8758	0.8364
13	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784	0.9658	0.9486	0.9261	0.8981
14	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897	0.9827	0.9726	0.9585	0.9400
15	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954	0.9918	0.9862	0.9780	0.9665
16	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980	0.9963	0.9934	0.9889	0.9823
17	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9984	0.9970	0.9947	0.9911
18	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997	0.9993	0.9987	0.9976	0.9957
19	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9995	0.9989	0.9980
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9991
21	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996
22	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999
23	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
24	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

CUMULATIVE POISSON PROBABILITIES

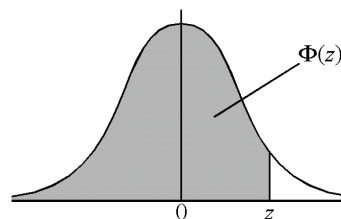
λ	10.00	11.00	12.00	13.00	14.00	15.00	16.00	17.00	18.00	19.00
$x = 0$	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009	0.0004	0.0002	0.0001	0.0000
5	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028	0.0014	0.0007	0.0003	0.0002
6	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076	0.0040	0.0021	0.0010	0.0005
7	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180	0.0100	0.0054	0.0029	0.0015
8	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374	0.0220	0.0126	0.0071	0.0039
9	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699	0.0433	0.0261	0.0154	0.0089
10	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185	0.0774	0.0491	0.0304	0.0183
11	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848	0.1270	0.0847	0.0549	0.0347
12	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676	0.1931	0.1350	0.0917	0.0606
13	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632	0.2745	0.2009	0.1426	0.0984
14	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657	0.3675	0.2808	0.2081	0.1497
15	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681	0.4667	0.3715	0.2867	0.2148
16	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641	0.5660	0.4677	0.3751	0.2920
17	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489	0.6593	0.5640	0.4686	0.3784
18	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195	0.7423	0.6550	0.5622	0.4695
19	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752	0.8122	0.7363	0.6509	0.5606
20	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170	0.8682	0.8055	0.7307	0.6472
21	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469	0.9108	0.8615	0.7991	0.7255
22	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673	0.9418	0.9047	0.8551	0.7931
23	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805	0.9633	0.9367	0.8989	0.8490
24	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888	0.9777	0.9594	0.9317	0.8933
25	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938	0.9869	0.9748	0.9554	0.9269
26	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967	0.9925	0.9848	0.9718	0.9514
27	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983	0.9959	0.9912	0.9827	0.9687
28	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991	0.9978	0.9950	0.9897	0.9805
29	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9989	0.9973	0.9941	0.9882
30	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9986	0.9967	0.9930
31	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9993	0.9982	0.9960
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9990	0.9978
33	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988
34	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994
35	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997
36	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
37	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
38	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

THE NORMAL DISTRIBUTION FUNCTION

If Z has a normal distribution with mean 0 and variance 1 then, for each value of z , the table gives the value of $\Phi(z)$, where

$$\Phi(z) = P(Z \leq z).$$

For negative values of z use $\Phi(-z) = 1 - \Phi(z)$.



z											ADD								
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359	4	8	12	16	20	24	28	32	36
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753	4	8	12	16	20	24	28	32	36
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141	4	8	12	15	19	23	27	31	35
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517	4	7	11	15	19	22	26	30	34
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879	4	7	11	14	18	22	25	29	32
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224	3	7	10	14	17	20	24	27	31
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549	3	7	10	13	16	19	23	26	29
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852	3	6	9	12	15	18	21	24	27
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133	3	5	8	11	14	16	19	22	25
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389	3	5	8	10	13	15	18	20	23
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621	2	5	7	9	12	14	16	19	21
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830	2	4	6	8	10	12	14	16	18
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015	2	4	6	7	9	11	13	15	17
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177	2	3	5	6	8	10	11	13	14
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319	1	3	4	6	7	8	10	11	13
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441	1	2	4	5	6	7	8	10	11
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545	1	2	3	4	5	6	7	8	9
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633	1	2	3	4	4	5	6	7	8
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706	1	1	2	3	4	4	5	6	6
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767	1	1	2	2	3	4	4	5	5
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817	0	1	1	2	2	3	3	4	4
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857	0	1	1	2	2	2	3	3	4
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890	0	1	1	1	2	2	2	3	3
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916	0	1	1	1	1	2	2	2	2
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936	0	0	1	1	1	1	1	2	2
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952	0	0	0	1	1	1	1	1	1
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964	0	0	0	0	1	1	1	1	1
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974	0	0	0	0	0	1	1	1	1
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981	0	0	0	0	0	0	0	1	1
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986	0	0	0	0	0	0	0	0	0

Critical values for the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that

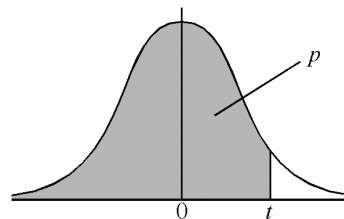
$$P(Z \leq z) = p.$$

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

CRITICAL VALUES FOR THE t -DISTRIBUTION

If T has a t -distribution with ν degrees of freedom then, for each pair of values of p and ν , the table gives the value of t such that:

$$P(T \leq t) = p.$$



p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$\nu=1$	1.000	3.078	6.314	12.71	31.82	63.66	127.3	318.3	636.6
2	0.816	1.886	2.920	4.303	6.965	9.925	14.09	22.33	31.60
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.21	12.92
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.894	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.689
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.660
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Mathematical notation

Examinations for the syllabus in this booklet may use relevant notation from the following list.

1 Set notation

\in	is an element of
\notin	is not an element of
$\{x_1, x_2, \dots\}$	the set with elements x_1, x_2, \dots
$\{x : \dots\}$	the set of all x such that ...
$n(A)$	the number of elements in set A
\emptyset	the empty set
\mathcal{E}	the universal set
A'	the complement of the set A
\mathbb{N}	the set of natural numbers, $\{1, 2, 3, \dots\}$
\mathbb{Z}	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
\mathbb{Z}^+	the set of positive integers, $\{1, 2, 3, \dots\}$
\mathbb{Z}_n	the set of integers modulo n , $\{0, 1, 2, \dots, n-1\}$
\mathbb{Q}	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
\mathbb{Q}^+	the set of positive rational numbers, $\{x \in \mathbb{Q} : x > 0\}$
\mathbb{Q}_0^+	set of positive rational numbers and zero, $\{x \in \mathbb{Q} : x \geq 0\}$
\mathbb{R}	the set of real numbers
\mathbb{R}^+	the set of positive real numbers, $\{x \in \mathbb{R} : x > 0\}$
\mathbb{R}_0^+	the set of positive real numbers and zero, $\{x \in \mathbb{R} : x \geq 0\}$
\mathbb{C}	the set of complex numbers
(x, y)	the ordered pair x, y
$A \times B$	the cartesian product of sets A and B , i.e. $A \times B = \{(a, b) : a \in A, b \in B\}$
\subseteq	is a subset of
\subset	is a proper subset of
\cup	union
\cap	intersection
$[a, b]$	the closed interval $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b)$	the interval $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b]$	the interval $\{x \in \mathbb{R} : a < x \leq b\}$
(a, b)	the open interval $\{x \in \mathbb{R} : a < x < b\}$
$y R x$	y is related to x by the relation R
$y \sim x$	y is equivalent to x , in the context of some equivalence relation

2 Miscellaneous symbols

$=$	is equal to
\neq	is not equal to
\equiv	is identical to or is congruent to
\approx	is approximately equal to
\cong	is isomorphic to
\propto	is proportional to
$<$	is less than
\leq	is less than or equal to
$>$	is greater than
\geq	is greater than or equal to
∞	infinity
$p \wedge q$	p and q
$p \vee q$	p or q (or both)
$\sim p$	not p
$p \Rightarrow q$	p implies q (if p then q)
$p \Leftarrow q$	p is implied by q (if q then p)
$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
\exists	there exists
\forall	for all

3 Operations

$a + b$	a plus b
$a - b$	a minus b
$a \times b, ab, a.b$	a multiplied by b
$a \div b, \frac{a}{b}, a / b$	a divided by b
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\prod_{i=1}^n a_i$	$a_1 \times a_2 \times \dots \times a_n$
\sqrt{a}	the positive square root of a
$ a $	the modulus of a
$n!$	n factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{Z}^+$ or
	$\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

4 Functions

$f(x)$	the value of the function f at x
$f: A \rightarrow B$	f is a function under which each element of set A has an image in set B
$f: x \mapsto y$	the function f maps the element x to the element y
f^{-1}	the inverse function of the function f
gf	the composite function of f and g which is defined by $gf(x) = g(f(x))$
$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as x tends to a

$\Delta x, \delta x$	an increment of x
$\frac{dy}{dx}$	the derivative of y with respect to x
$\frac{d^n y}{dx^n}$	the n th derivative of y with respect to x
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ..., n th derivatives of $f(x)$ with respect to x
$\int y \, dx$	the indefinite integral of y with respect to x
$\int_a^b y \, dx$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
$\frac{\partial V}{\partial x}$	the partial derivative of V with respect to x
\dot{x}, \ddot{x}, \dots	the first, second, ... derivatives of x with respect to t

5 Exponential and logarithmic functions

e	base of natural logarithms
$e^x, \exp x$	exponential function of x
$\log_a x$	logarithm to the base a of x
$\ln x, \log_e x$	natural logarithm of x
$\lg x, \log_{10} x$	logarithm of x to base 10

6 Circular and hyperbolic functions

$\left. \begin{array}{l} \sin, \cos, \tan \\ \operatorname{cosec}, \sec, \cot \end{array} \right\}$	the circular functions
$\left. \begin{array}{l} \sin^{-1}, \cos^{-1}, \tan^{-1} \\ \operatorname{cosec}^{-1}, \sec^{-1}, \cot^{-1} \end{array} \right\}$	the inverse circular functions
$\left. \begin{array}{l} \sinh, \cosh, \tanh \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \sinh^{-1}, \cosh^{-1}, \tanh^{-1} \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \operatorname{coth}^{-1} \end{array} \right\}$	the inverse hyperbolic functions

7 Complex numbers

i	square root of -1
z	a complex number, $z = x + iy = r(\cos \theta + i \sin \theta)$
$\operatorname{Re} z$	the real part of z , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of z , $\operatorname{Im} z = y$
$ z $	the modulus of z , $ z = \sqrt{x^2 + y^2}$
$\arg z$	the argument of z , $\arg z = \theta, -\pi < \theta \leq \pi$
z^*	the complex conjugate of z , $x - iy$

8 Matrices

\mathbf{M}	a matrix \mathbf{M}
\mathbf{M}^{-1}	the inverse of the matrix \mathbf{M}
\mathbf{M}^T	the transpose of the matrix \mathbf{M}
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix \mathbf{M}

9 Vectors

\mathbf{a}	the vector \mathbf{a}
\overrightarrow{AB}	the vector represented in magnitude and direction by the directed line segment AB
$\hat{\mathbf{a}}$	a unit vector in the direction of \mathbf{a}
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
$ \mathbf{a} , a$	the magnitude of \mathbf{a}
$ \overrightarrow{AB} , AB$	the magnitude of \overrightarrow{AB}
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of \mathbf{a} and \mathbf{b}
$\mathbf{a} \times \mathbf{b}$	the vector product of \mathbf{a} and \mathbf{b}

10 Probability and statistics

A, B, C , etc.	events
$A \cup B$	union of the events A and B
$A \cap B$	intersection of the events A and B
$P(A)$	probability of the event A
A'	complement of the event A
$P(A B)$	probability of the event A conditional on the event B
X, Y, R , etc.	random variables
x, y, r , etc.	values of the random variables X, Y, R , etc.
x_1, x_2, \dots	observations
f_1, f_2, \dots	frequencies with which the observations x_1, x_2, \dots occur
$p(x)$	probability function $P(X=x)$ of the discrete random variable X
p_1, p_2, \dots	probabilities of the values x_1, x_2, \dots of the discrete random variable X
$f(x), g(x), \dots$	the value of the probability density function of a continuous random variable X
$F(x), G(x), \dots$	the value of the (cumulative) distribution function $P(X \leq x)$ of a continuous random variable X
$E(X)$	expectation of the random variable X
$E(g(X))$	expectation of $g(X)$
$\text{Var}(X)$	variance of the random variable X
$G(t)$	probability generating function for a random variable which takes the values $0, 1, 2, \dots$
$B(n, p)$	binomial distribution with parameters n and p
$\text{Geo}(p)$	geometric distribution with parameter p
$\text{Po}(\lambda)$	Poisson distribution with parameter λ
$N(\mu, \sigma^2)$	normal distribution with mean μ and variance σ^2
μ	population mean
σ^2	population variance
σ	population standard deviation
\bar{x}, m	sample mean
$s^2, \hat{\sigma}^2$	unbiased estimate of population variance from a sample,
	$s^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$
ϕ	probability density function of the standardised normal variable with distribution $N(0, 1)$
Φ	corresponding cumulative distribution function

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